PROCESSING OF STOCHASTIC SAMPLED DATA IN LASER DOPPLER ANEMOMETRY

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ABSTRACT

The laser Doppler anemometry (LDA) as a non-intrusive technique of velocity measurements is widely used in the study of turbulent flow fields. The computation of statistical functions of the measured velocity time series presents problems, since, by nature of the technique, the data set is stochastically sampled in time. After a brief review of the LDA measurement principles the essential properties of LDA data sets will be explained and a review of the methods to handle the special sampling features of the data will be given. Recent developments will be presented for the estimation of autocorrelation functions and turbulence spectra.

1. LDA TECHNIQUE

The laser Doppler anemometry (LDA) is a widely accepted tool for fluid dynamic investigations in gases and liquids and has been used as such for more than three decades. It is a well-established technique that gives information about flow velocity. Its non-intrusive nature and directional sensitivity make it very suitable for applications with recirculating flow, chemically reacting or high-temperature media and rotating machinery, where physical sensors are difficult or impossible to use. It requires tracer particles in the flow.

The basic configuration of an LDA system [6, 35] consists of (figure 1) a continuous wave laser, transmitting optics, receiving optics and a photodetector.

The laser beam is split into two beams and the focusing lens forces the two beams to intersect. In the region of intersection the two laser beams interfere to produce light intensity variations leading to parallel planes of constant intensity with a Gaussian envelope (figure 2). The planes have the constant distance Δx given through the wave length of the laser light and the angle between the intersecting beams. The envelope forms a prolate ellipsoid of constant amplitudes. A typical size of the measurement volume is $40 \times 40 \times 200\,\mu$m that defines the spatial resolution of the LDA system.

The photodetector receives light scattered from tracer particles moving through the measurement volume and converts the light intensity into electrical current, the burst signal (figure 3). The burst frequency is proportional to the velocity component perpendicular to the bisector of the two laser beams. The signal processing removes noise from the signal and extracts the burst frequency and hence the velocity information.

Another way to interpret the principle of a LDA system is to presume that the laser beams are scattered separately by the tracer particle. For a moving particle the scattered light is Doppler shifted, hence the name of the technique. The frequency shift is different for both laser beams. The scattered light interferes on the surface of the photodetector, where the Doppler frequency can be measured.

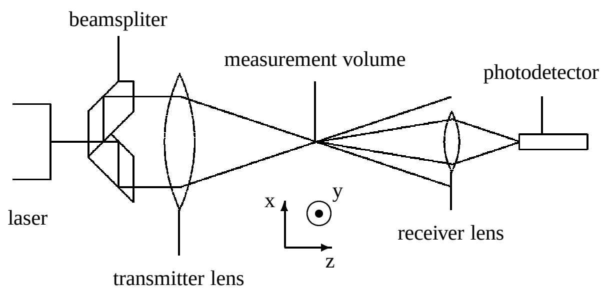

Figure 1: Set-up of a laser Doppler anemometer

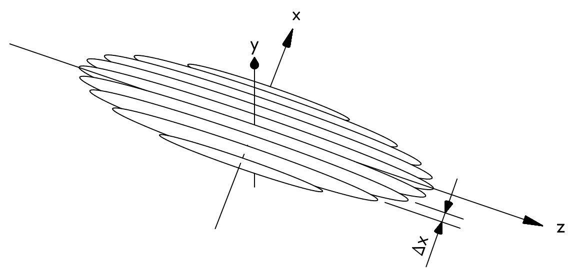

Figure 2: The LDA measurement volume

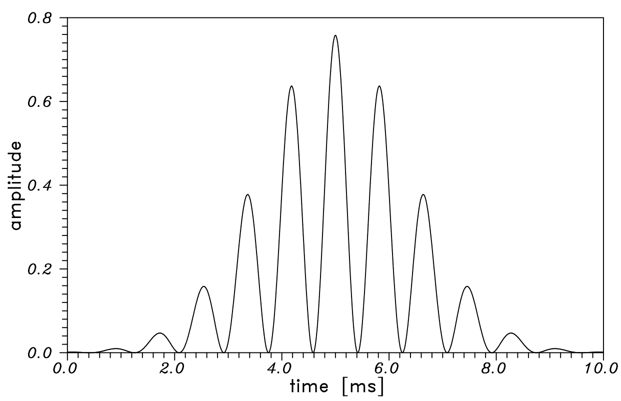

Figure 3: LDA burst signal

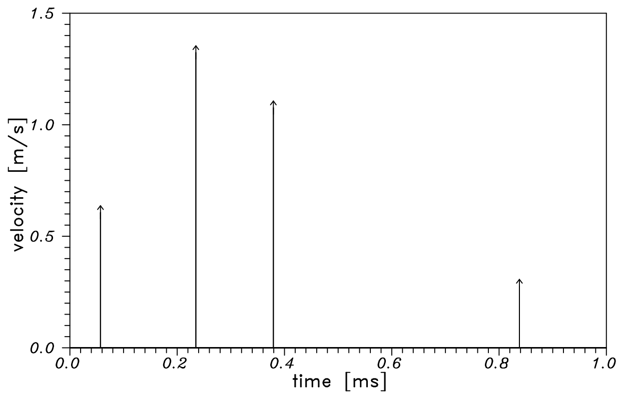

Figure 4: Stochastic sampling with a laser Doppler anemometer

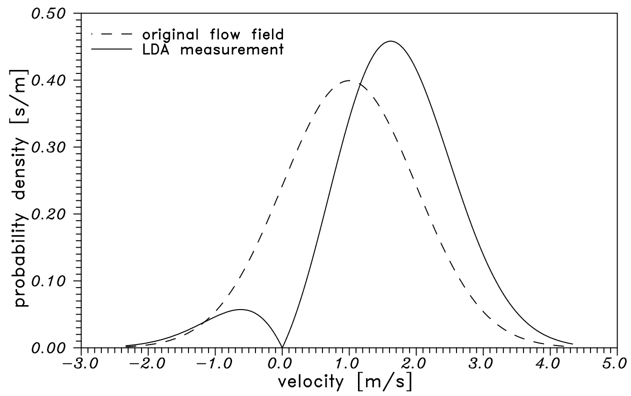

Figure 5: Velocity distribution of the original flow field compared to the data set

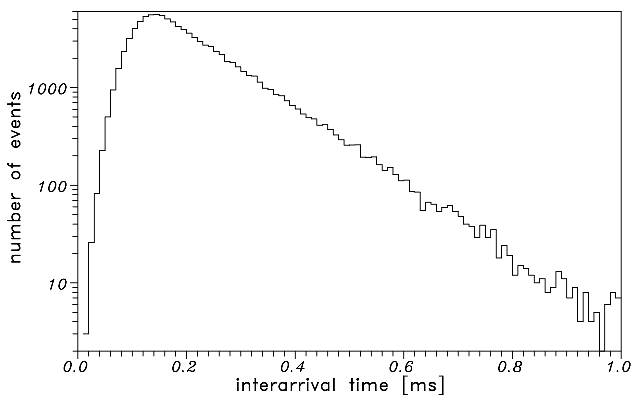

Figure 6: Distribution of interarrival times

2. LDA DATA SET

The LDA technique gives a transient history of velocity values derived from individual particles that cross the measurement volume. For each velocity measurement there exists an arrival time of the corresponding particle. The LDA data set represents a series of time-velocity pairs.

2.1. Stochastic sampling

The most important influence to the characteristics of the LDA data set is given through the dependence on tracer particles. In [7, 9] investigations on sampling statistics are presented. Presuming an equal particle distribution in space with a constant concentration c_s, the intervals Δt between the measurements are distributed exponentially

$$p(\Delta t) = \dot{n} e^{-\dot{n}\Delta t} \tag{1}$$

with the mean data rate $\dot{n}$ (figure 4).

2.2. Correlation between velocity and data rate

For non-constant velocities the data rate is not constant in spite of a constant particle concentration c_s. Presuming a constant spatial distribution of tracer particles the fluid or gas volume that passes the measurement volume within a given time is proportional to the velocity and hence, the number of measurements is proportional to the velocity as well. This leads to a frequent occurrence of high velocity measurements and to a distorted velocity distribution (figure 5).

Furthermore, the data rate depends on the size of the measurement volume. It is proportional to the projection area $A_\perp$ of the measurement volume normal to the flow velocity vector. Because of the significant prolate shape of the measurement volume the size of the projection depends on the direction of the flow. An expression of the variable data rate is given through

$$\dot{n} = c_s A_\perp(\vec{u})\,|\vec{u}| \tag{2}$$

which describes the correlation of the data rate and the velocity [21]. Consequently, the sampling scheme depends on the measured velocity value.

2.3. Noise

The optical system, the velocity gradients within the measurement volume, the temporal and amplitude resolution of the detector and the signal processing lead to random deviations of the velocity measurement and thus to a noise component in the LDA data set [5, 13].

2.4. Processor delay

The velocity measurements ensue normally from single particles. If two particles enter the measurement volume within a short time interval the burst signals overlap. Because of the phase difference of the bursts a phase drift can be seen within the double burst signal. Therefore, the burst frequency cannot be derived exactly. These multi-burst signals are detected by pre-processors that reject them from the data stream. This leads to a significant underrepresentation of small interarrival times (figure 6).

3. LDA DATA PROCESSING

The stochastic sampling given through the observed process with its complex dependence on the velocity vector requires specific methods for the data processing. There are two main tasks of LDA data processing:

1. the adequate reconstruction of the velocity as a function of the time and
2. the estimation of statistical values and functions like the mean, the variance, the autocorrelation function (ACF) or the power spectral density (PSD) of the flow velocity fluctuations.

The reconstruction of the continuous velocity function from irregularly sampled data has a tradition at the SampTA conference. The individuality of the reconstruction from LDA data sets is determinated by the extremely low data rate with significant power in the original signal above the mean data rate, the variable data rate, the dependency of the sampling rate on the measurement value, the variable sampling distribution and the noise which can be as large as the original velocity signal. A powerful algorithm for relatively high data rates is given in [20].

In many cases the exact velocity function is not required, only the statistics of the flow field. To derive the flow field statistics from an LDA data set two methods are possible:

1. the direct estimation from the LDA data set using the information of the sampling statistics and
2. the reconstruction of the continuous velocity function, possibly with an equidistant resampling with respect to the digital signal processing and statistic's estimation from the reconstructed function.

In contrast to the normal reconstruction task, here the only requirement is the preservation of the statistics and the result can therefore look strange. Even so, it is possible to use any reconstruction method if there is an appropriate transform to correct the statistics calculated from the reconstructed function.

Because of the complexity of the sampling statistics a complete mathematical description has not yet been derived. Instead, several groups of researchers have attempted to optimize individual algorithms for specific applications. The goal is to minimize the bias and the estimator's variance for special conditions. The second goal is to get robust algorithms for drifting process parameters. Because of the great variaty of applications many different algorithms for statistical analysis exist.

The pioneering work in spectral analysis from LDA data resulted in two estimators, the *slotting technique* [16, 27, 11, 28, 29] and a *direct transform* [12]. But the problem of velocity bias [21, 8] focused attention on simpler velocity statistics.

The cause of the velocity bias is the dependence of the data rate on the velocity (section 2.2). The distorted velocity distribution of the measured LDA data (figure 5) exhibits other statistics than the original flow field. The mean and the variance estimated from the LDA data set through

$$\hat{m}_u = \frac{1}{N}\sum_{i=1}^{N} u_i \tag{3}$$

$$\hat{\sigma}_u^2 = \frac{1}{N-1}\sum_{i=1}^{N} (u_i - \hat{m}_u)^2 \tag{4}$$

with N velocity samples u_i have large deviations (figure 7). The hat on top of the sign indicates the estimation of the statistical value.

With a weighting technique [4] with individual weights w_i for each measured velocity value u_i

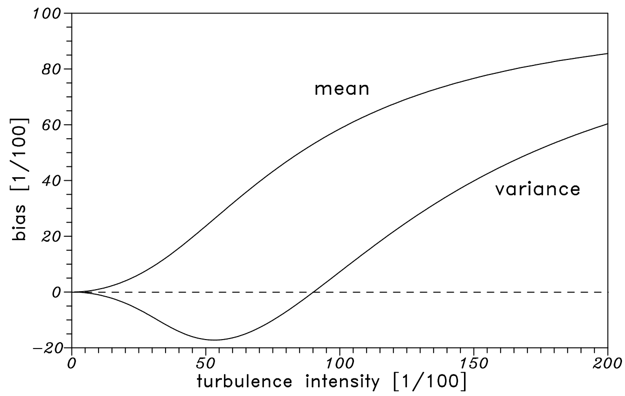

Figure 7: The bias of mean and variance estimation from LDA data without a weighting algorithm (turbulence intensity= $\sqrt{\sigma_u^2/m_u}$)

$$\hat{m}_u = \frac{\sum_{i=1}^{N} u_i w_i}{\sum_{i=1}^{N} w_i} \tag{5}$$

$$\hat{\sigma}_u^2 = \frac{\sum_{i=1}^{N} (u_i - \hat{m}_u)^2 w_i}{\sum_{i=1}^{N} w_i} \tag{6}$$

the results are much more reliable. Several methods to derive the weights from the LDA data set were developed, e.g. the velocity weighting [21], the arrival time weighting [2] and the transit time weighting [14].

These algorithms were investigated [10, 26], so that their advantages and disadvantages could be recovered for different conditions. The one-dimensional velocity weighting ($w_i = 1/u_i$) is suitable only for one-dimensional flows. In the case of high turbulence intensity or shear stesses the three-dimensional velocity vector is necessary ($w_i = 1/|\vec{u}_i|$). This requires a very expensive three-dimensional LDA system. Nevertheless, the velocity weighting is very noise sensitive, so that it is suitable only for very high burst signal qualities.

The transit time weighting ($w_i = \tau_i$) uses the time the tracer particle needs to pass the measurement volume. With a good estimate of the transit time τ_i this weighting method is exact for constant particle concentrations even for three-dimensional turbulence. But the estimation of the transit time from a LDA burst is very complicated. The time is quantified with the period of the Doppler frequency, the signal can have a large noise level and the effective size of the measurement volume depends on the particle size.

The most robust weighting technique is the arrival time weighting ($w_i = 1/\Delta t_i$) using the interarrival time between two particles $\Delta t_i = t_i - t_{i-1}$. The effectivity of the arrival time weighting depends on the data rate. For high data rates the velocity bias disappears. With lower data rates the effectivity becomes smaller, but the direction of operation is always correct. Furthermore, this is the only weighting method that works with non-constant particle concentrations.

In the mid to late eighties a gradual rekindling of interest in dynamic statistics like ACF or PSD took place as investigators tried to use them as tools to study the small scales of turbulence. Comparative studies [30, 32] indicated that the early estimators

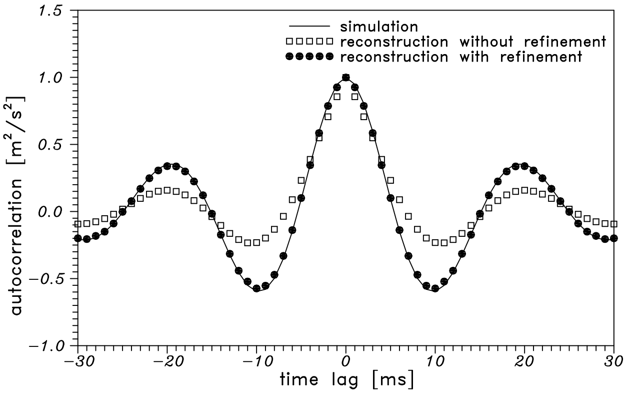

Figure 8: The effect of the refinement for the sample-and-hold reconstruction

possessed a high degree of variance and a susceptibility to velocity bias. Even the age old sample-and-hold reconstruction of LDA data led to a filtered noise effect, which obscures the high frequency portion of the spectrum [1].

The results of statistical estimations from reconstructed LDA data sets were found to be very sensitive to the reconstruction method used and thus investigations to find the best method of reconstruction [34, 31, 17, 18, 15] were started. A comparison of several reconstruction methods [22] leads to the conclusion that all have a similar behavior. The data rate was found to be the main parameter, similar to the findings for the sample-and-hold reconstruction in [1]. For high data rates the results are reliable with marginal differences between the reconstruction schemes. For low data rates all reconstruction methods have a low pass character, known as the *data rate filter*, that suppresses the higher frequencies and leads to aliasing errors, where the different reconstruction schemes have their individual filter characteristics.

In [1] a mathematical description of the filter function is derived for the sample-and-hold reconstruction, that gives an expression of the ACF calculated from the reconstructed velocity function in terms of the true ACF. A similar expression for the sample-and-hold reconstruction in combination with an equidistant resampling is given in [23]. The equidistant resampling leads to a linear data rate filter that can be inverted. The inverse filter can be used to correct the ACF derived from the reconstruction.

The sample-and-hold reconstructed function $u^{(r)}(t)$ is resampled with equidistant intervals $\Delta\tau$. The ACF of the new data set $u_i^{(r)} = u^{(r)}(i\Delta\tau)$ is given through

$$\hat{R}_k^{(r)} = \hat{R}^{(r)}(k\Delta\tau) = \frac{1}{N^{(r)} - |k|} \sum_{i=1}^{N^{(r)}-|k|} u_i^{(r)} u_{i+|k|}^{(r)} \tag{7}$$

with the number $N^{(r)}$ of data points in the reconstructed and resampled data set. The filtered ACF

$$\hat{R}_k^{(f)} = \begin{cases} \hat{R}_0^{(r)} & \text{for } k = 0 \\ (2c_f + 1)\hat{R}_k^{(r)} - c_f\left(\hat{R}_{k-1}^{(r)} + \hat{R}_{k+1}^{(r)}\right) & \text{for } k \neq 0 \end{cases} \tag{8}$$

with

$$c_f = \frac{e^{-\dot{n}\Delta\tau}}{(1 - e^{-\dot{n}\Delta\tau})^2} \tag{9}$$

leads to a refined estimation, with the filter parameter c_f that depends only on the mean data rate $\dot{n}$. Nevertheless, the noise in the LDA data set leads to a systematic error in the ACF and the PSD. A method to estimate the noise power and to correct the estimation is given in [25].

Figure 8 shows the ACF estimates using a sample-and-hold reconstruction with and without that refinement. The reconstruction without refinement shows significant systematic errors. The refinement is able to correct the effect of the particle rate filter completely.

The use of more complex reconstruction methods is possible in principle, but the correction filter becomes unreasonably.

The second method with remarkable improvement in the last few years is the slotting technique. In principle the arrival times are quantified to get a quasi equidistant sampled data set with large intervals without measurements. To derive the ACF the interval between every two measurements is splitted into small bins (slots) of width $\Delta\tau$. The ACF is given through

$$\hat{R}(k\Delta\tau) = \frac{\sum_{\substack{i;j=1 \\ i\neq j}}^{N} u_i u_j b_k(t_j - t_i)}{\sum_{\substack{i;j=1 \\ i\neq j}}^{N} b_k(t_j - t_i)} \tag{10}$$

with the mask function

$$b_k(\tau) = \begin{cases} 1 & \text{for } \left(k - \frac{1}{2}\right) \leq \frac{\tau}{\Delta\tau} < \left(k + \frac{1}{2}\right) \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

The use of cross products only ($i \neq j$) leads to an estimation that has no bias through the noise, because it is independent for each velocity sample.

In the last few years two important improvements were developed for that technique, the *local normalization* [33] and the *fuzzy slotting technique* [24].

The local normalization uses an alternative normalization to calculate correlation coefficients $\rho(\tau) \stackrel{\text{def}}{=} R(\tau)/R(0)$ through

$$\hat{\rho}(k\Delta\tau) = \frac{A}{\sqrt{BC}} \tag{12}$$

with

$$A = \sum_{\substack{i;j=1 \\ i\neq j}}^{N} u_i u_j b_k(t_j - t_i) \tag{13}$$

$$B = \sum_{\substack{i;j=1 \\ i\neq j}}^{N} u_i^2 b_k(t_j - t_i) \tag{14}$$

$$C = \sum_{\substack{i;j=1 \\ i\neq j}}^{N} u_j^2 b_k(t_j - t_i) \tag{15}$$

Instead of the normal norm factor σ_u^2 this technique uses only these velocity values for the estimation of $R(0)$ that are used for the estimation $R(\tau)$. The ACF can be found with the factor $\hat{\sigma}_u^2$ from equation (4) or (6).

$$\hat{R}(k\Delta\tau) = \frac{\hat{\sigma}_u^2 A}{\sqrt{BC}} \tag{16}$$

The effect of this technique is a decreased variability of the estimate especially for large correlation coefficients.

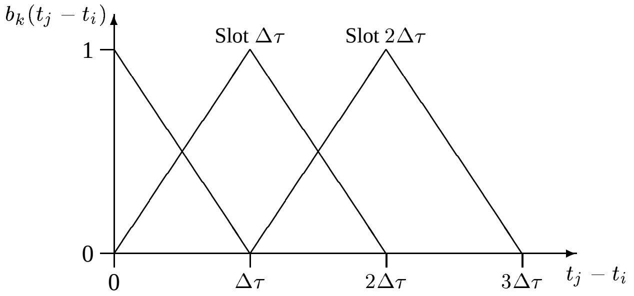

Figure 9: Fuzzy Slotting Technique

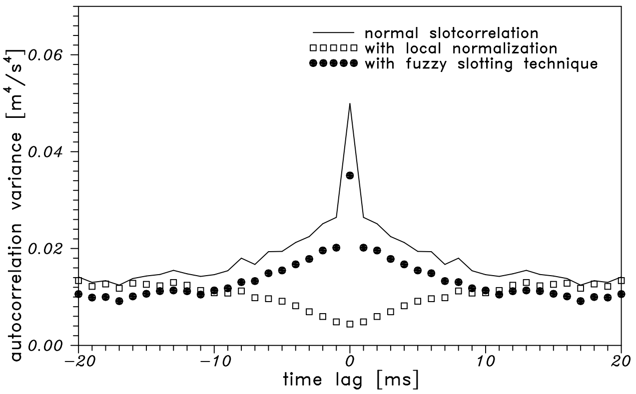

Figure 10: The reduction of estimation's variability through the *local normalization* and the *fuzzy slotting technique*

The fuzzy slotting technique uses a lag products weighting scheme (figure 9) defined as

$$b_k(\tau) = \begin{cases} 1 - \left|\frac{\tau}{\Delta\tau} - k\right| & \text{for } \left|\frac{\tau}{\Delta\tau} - k\right| < 1 \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

which allows lag products to contribute to two slots simultaneously and weights lag products that lie close to the slot center more heavily. This leads to a reduced variability of the estimate and at the same time a reduced bias through the averaging within the slots.

In figure 10 the estimator's variability is shown for the local normalization and for the fuzzy slotting technique in comparison to the normal slotting technique. While the local normalization reduces the variability only for large correlation coefficients close to the time lag zero with very high effectivity the fuzzy slotting technique is not as effective but it reduces the variablity for any time lag.

These results are confirmed by the recently performed *benchmark tests* [3] of spectral estimation from LDA data sets. Here the idea was born to merge both techniques [19], producing a more powerful estimator.

A further development of this technique is the implementation of the weighting method with

$$A = \sum_{\substack{i;j=1 \\ i\neq j}}^{N} u_i u_j w_i w_j b_k(t_j - t_i) \tag{18}$$

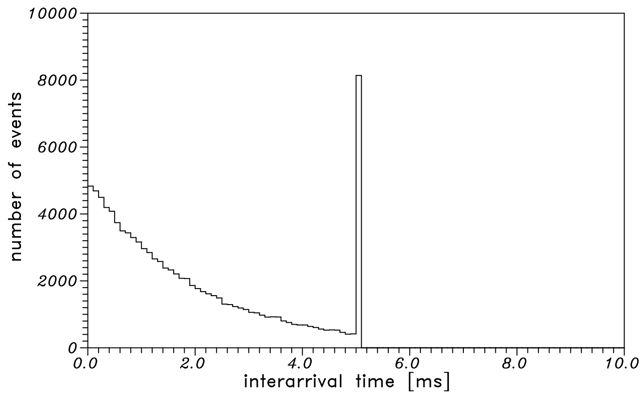

Figure 11: Distribution of the interarrival time $t_i - t_{i-1}$ for the slotcorrelation with $k\Delta\tau = 5\,\mathrm{ms}$ (simulation without processor delay)

$$B = \sum_{\substack{i;j=1 \\ i\neq j}}^{N} u_i^2 w_i w_j b_k(t_j - t_i) \tag{19}$$

$$C = \sum_{\substack{i;j=1 \\ i\neq j}}^{N} u_j^2 w_i w_j b_k(t_j - t_i) \tag{20}$$

and the variance estimate $\hat{\sigma}_u^2$ from equation (6) to overcome the velocity bias.

The use of the velocity weighting or the transit time weighting is possible like for the statistical moments. Instead of the normal arrival time weighting the *forward-backward arrival time weighting* should be prefered with

$$\begin{array}{llll} w_i = t_i - t_{i-1} & \text{and} & w_j = t_{j+1} - t_j & \text{for} \quad j > i \\ w_i = t_{i+1} - t_i & \text{and} & w_j = t_j - t_{j-1} & \text{for} \quad j < i \end{array} \tag{21}$$

because of the dependence of the interarrival time distribution on the calculated time lag $k\Delta\tau$ (figure 11).

4. CONCLUSIONS

Due to its the physical working principle, the data processing in the laser Doppler anemometry is an application of stochastic sampling. For many cases the adequate reconstruction of the flow velocity function is a secondary task. The main task of LDA data processing is the calculation of flow statistics. That can be done directly from the data set containing the information about the sampling statistics or via a reconstruction that preserves the statistics or a reconstruction with a subsequent refinement that corrects the statistics calculated from the reconstructed function.

The uniqueness of LDA data processing is the very low data rate, that is even depending on the velocity itself, the special sampling characteristics without an upper limit of intervals between the samples, and the possible drifting process parameters in turbulent flow fields.

The sample-and-hold reconstruction with its refinement and direct spectral noise removal [25] and the slotcorrelation with local normalization and fuzzy slotting technique [19] in combination with the forward-backward arrival time weighting are powerful estimators of the autocorrelation, that have a small estimation variability and that are very stable for several applications.

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Corrections

page 3, caption of figure 7:	turbulence intensity $= \sqrt{\sigma_u^2}/m_u$
page 3, right column, last but one paragraph:	$w_i = \Delta t_i$