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MEASUREMENT OF LAGRANGIAN ACCELERATION IN TURBULENT FLOWS USING THE LASER DOPPLER TECHNIQUE

The use of the laser Doppler technique for measuring Lagrangian acceleration with a high spatial resolution is introduced. The requirements in system alignment and accuracy of signal processing are estimated. Specifications of the optical design and the signal processing are given.

LAGRANGIAN ACCELERATION, LASER DOPPLER

1. INTRODUCTION Based on the scaling theory of Kolmogorov, the statistics of Lagrangian acceleration can be predicted. From the fluid mechanics point of view this is interesting, since the left-hand side of the Navier-Stokes equations are formulated in terms of fluid acceleration.

The measurement of Lagrangian acceleration can be realized by observing the material derivative of the velocity. In principle, all measurement techniques can be used which utilize tracer particles. In recent years, several new measurement techniques have been developed based on particle tracking using imaging systems [1,2] or spatial filters [6,7]. Due to the limited spatial resolution of such systems, the trajectory of a specific particle must be observed over a rather large distance to deduce acceleration.

The laser Doppler technique offers much higher spatial resolution and can therefore be considered as an potential alternative [3,4,5]. Here, the particle acceleration is obtained as the derivative of the velocity for each individual particle crossing the measurement volume of the laser Doppler system. Unfortunately, due to the small size of the measurement volume, the actual measured change of signal frequency is very small. Assuming a measurement volume of 100µm diameter and a mean velocity of 10m/s, the velocity of a particle accelerated with 1000m/s^2, which is the usual order of particle acceleration in a turbulent flow, leads to a velocity change of 0.01m/s within the measurement volume. Because this is only 0.1% of the mean velocity, the requirements in measurement accuracy of the system are very high.

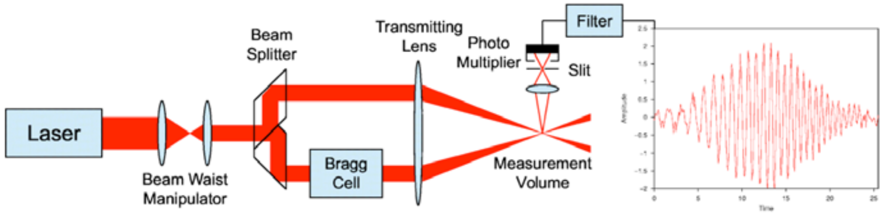

Fig. 1: Optical set-up and measurement signal

2. OPTICAL DESIGN The obtainable accuracy of frequency estimation using a laser Doppler system is limited by both systematic and random errors. Systematic errors are mainly introduced by the divergence of the fringe system, which occurs if the beam waists do not coincide with the beam intersection point. An accurate alignment of the optical system can significantly reduce this influence. Therefore, the basic optical set-up is a modified laser Doppler system, where all components can be aligned separately. However, due to the non-planar light waves in the measurement volume, some fringe divergence remains. The problem can be reduced only by increasing the size of the measurement volume by means of beam adjustment before the transmitting lens. To achieve this, a beam waist manipulator has been assembled into the beam to reduce the waist diameter before the transmitting lens, which then increases the Rayleigh length and the accuracy of the fringe system (fig. 1). Furthermore, a slit is used in the receiving path, to cut out a small fraction of the measurement volume from the center with a minimum fringe divergence.

Unfortunately, this also reduces the available light intensity, and reduces the signal-to-noise ratio, hence increasing random errors. Since the measured effect is rather small, also the random errors limit the maximum accuracy of such a system. To realize the required accuracy to obtain acceleration statistics, the laser Doppler system must be optimized towards a balance between systematic and random errors. Furthermore, all components of the system must be realized as accurately as possible.

3. SIGNAL PRE-PROCESSING Principally, the acceleration of the particle can be calculated from two velocity estimates during the passage of the particle through the measurement volume. Since the signal frequency corresponds to the particle velocity, the frequency change yields the acceleration. However, the signal processing must deliver bias-free estimates with the lowest possible estimation variance to fulfil the accuracy requirements. In the past, several direct and model-based estimators have been developed [5]. However, one model-based method has been found to be very reliable, it yields accurate velocity and acceleration estimates, close to the Cramer-Rao lower bound. Furthermore, it is very robust under the given conditions of non-linear amplitude variations.

The efficiency of the estimation depends on the signal pre-filtering. The frequency shift, realized by the Bragg cell, yields a signal close to 40MHz, which requires at least a bandwidth of about 100MHz of the digitizer. Alternatively, the signal can be shifted down to the base band. In the latter case, two orthogonal signals can be obtained by using two reference signals, shifted by 90 degrees. This is principally superior to the option of calculating the orthogonal signal based on a Hilbert transform, because the two signals have independent current and quantization noise and, therefore, more information content. However, the signals have the same source of photon noise, since the signal in the 40MHz band has been shared. Therefore, the signal in the 40MHz band and the two orthogonal signals in the base band are equivalent, providing an accurate analogue noise filtering, which cuts the noise power above the Nyquist frequency of the digitizer. The following signal processing routine can be used either in 40MHz or in the base band. Since it makes use of the two orthogonal signals, the phase shifted signal in the 40MHz band must be calculated by a Hilbert transform.

4. MODEL-BASED SIGNAL PROCESSING Model-based estimators principally use *a priori* information about the signal to improve the accuracy. By fitting a model signal to the measurement, the model parameters yield the required quantities. Since the amplitude of the Doppler signal can be distorted by several effects of the laser Doppler system and the absolute signal phase is random, the signal model is reduces to

$$s_i = \cos\left[\frac{\pi a}{\Delta x} t_i^2 + \frac{2\pi v}{\Delta x} t_i\right] + \mathrm{j}\sin\left[\frac{\pi a}{\Delta x} t_i^2 + \frac{2\pi v}{\Delta x} t_i\right] \qquad (1)$$

with the velocity v and the Lagrangian acceleration a to be estimated. Appropriate modifications are necessary to obtain the midpoint velocity or to include the frequency shift. The two parameters are optimized in such a way, that the correlation between the measured and the model signal is maximized.

$$R = \left|\sum s_i \cdot \hat{s}_i\right| \Rightarrow \max \qquad (2)$$

Note that both signals are complex, which makes the result independent of the signal phases. Additionally, the high amplitude signal parts, which correspond to more available information, are weighted more heavily than the noisy low-amplitude parts. Performance tests of this estimator have shown that the Cramer-Rao lower bound is almost reached and, that the estimator works reliably even for strongly distorted signal envelopes.

5. CONCLUSION The laser Doppler system is a potential alternative to imaging systems for measuring Lagrangian acceleration. Adaptations of the laser Doppler system are required to balance the systematic errors caused by fringe divergence and random errors, caused by several noise sources. With the set-up introduced in this study and the model-based signal processing, a final resolution of 1000m/s^2 can be realized, which is the usual order of particle acceleration in a turbulent flow.

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