LDA Signal Reconstruction: Application to Moment and Spectral Estimation

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Abstract

The reconstruction of an LDA signal refers to the interpolation of the measured velocity values obtained at random times, resulting in a velocity signal continuous in time. In particular, this paper deals with the bias and variance of several moment and spectral estimators based on reconstructed LDA signals. In addition to conventional reconstruction techniques, two more recent methods are investigated, including *projection onto convex sets* and *fractal reconstruction*. Both simulations and experiments have been used to evaluate the suitability of the various reconstruction techniques as a function of the flow and seeding parameters.

1 Introduction

Flow velocity information is obtained with an LDA at irregular time intervals, corresponding to random particle arrivals in the control volume. Whereas the information content of data obtained by equidistant sampling of a continuous process is well defined by Shannon theorem [16], there currently exists no equivalent statement for randomly sampled signals. Furthermore, the probability density function of the sample intervals is dependent on the instantaneous velocity magnitude. These properties of LDA signals must be considered in formulating moment and spectral estimators to avoid bias errors [2, 5, 7].

Considerable attention has been directed to extracting moment and spectral information from LDA signals, usually with the intuitively acceptable conclusion that a higher mean particle rate will lead to better estimates. Parameter estimation customarily takes one of two approaches. Either estimates are based directly on the available velocity samples, their arrival times and possibly further information such as residence time; or an interpolation of the velocity signal between the measured values is performed, followed by an estimation based on the reconstructed signal, often using an equidistant sampling of the reconstructed signal.

One may ask why such a myriad of data processing approaches exist and what the motivation for pursuing novel approaches can be. Three interesting situations can be cited for which improvements can be envisioned.

1. At present, direct spectral estimation of LDA signals is basically a trade-off between bandwidth and variability[10]. At higher frequencies, spectral estimates become less certain. This behaviour is well documented and is not likely to change in principle even with improved estimators [15, 17, 20]. However, the estimator variability may decrease for some estimators, especially if physically plausible information regarding the spectrum of turbulent fluctuations can be entered into the estimation *a priori*. A reconstruction model may be capable of achieving this and thus extend the bandwidth or improve the variability of the estimate.

2. There exist several frequently encountered measurement situations in which the seeding density is not homogeneous and may be correlated with the measured velocity. Mixing layers originating from two different flow sources, or combustion systems in which the detection of the seeding particles is influenced by the combustion are examples. In such situations unbiased moment or spectral estimators are difficult to formulate and here again, a reconstruction of the signal may offer an acceptable alternative [21].

3. The third situation concerns spectral estimation of short time records, necessary when analysing transient flows. One such example is length scale estimation in an internal combustion engine. The validity of Taylor's hypothesis put aside, rough length scale estimates are often only available through a spectral analysis of single point velocity measurements in an engine. Clearly, length scales vary dramatically throughout the cycle so that an estimation must be made on relatively short time records. Conventional spectral methods are unsuited to this task, as is well documented by the increasing number of methods developed for speech processing [11].

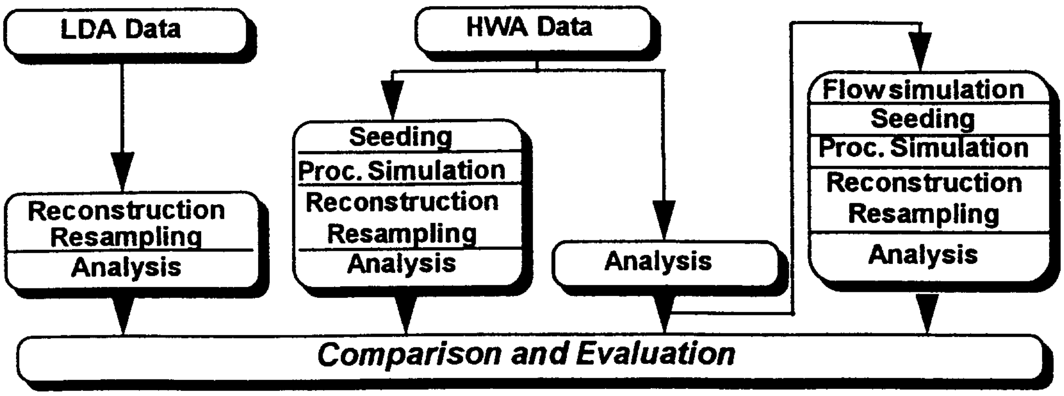

Figure 1: Schematic representation of how experimental data was used to verify results of the simulation program.

The present paper does not attempt to derive a theoretical statement about what information is actually contained in LDA data. Rather it uses an empirical approach to compare various suggestions which have been put forward in the literature, and to investigate under what conditions these estimation schemes perform well. But even with this more modest approach, it is essential that the evaluation of the result is accurate enough to recognize possible differences or improvements between estimators. This has been achieved by a combination of simulation and experiment as summarized in Fig. 1 and described briefly below.

The study of reconstructed LDA signals builds on previous work involving the simulation of LDA data of given statistical properties using a trivariant autoregressive process [8, 9]. The simulated LDA data can then be processed and compared with the known signal properties. In the present study this approach has been refined by also obtaining data from a flowfield. Both LDA and hot-wire data (HWA) were taken from various positions in a flow. Direct comparisons could be obtained between an analysis of the LDA data and the continuous HWA data. Furthermore, the HWA data could be used both directly and indirectly to simulated the LDA data and to obtain a further reasssurance that the simulation results were realistic. Finally, purely simulated results provided the flexibility to investigate situations in which an equivalent experiment would be difficult to perform.

2 Description of Reconstuction Models

This section is purposely kept concise because the models used are taken from the literature and described elsewhere in detail [14].

2.1 Polynomial Interpolation

A zero order and first order polynomial have been used for interpolation. The zero order interpolation corresponds to the well-documented sample and hold (S+H) or arrival time weighted estimator[1, 6, 23]. The first order interpolation will be referred to as a **linear** interpolation.

2.2 'Shannon' Reconstruction (SR)

First introduced by Clark *et al.*[4] and applied to LDA data by Veynante and Candel[21], this approach stretches the time axis such that the velocity samples lie at equally spaced time intervals. Shannon reconstruction[16] can then be applied to interpolate intermediate points. After the stretched-signal reconstruction, the inverse of the stretching transformation can be used to resample the signal at the times corresponding to equal intervals prior to the stretching. This approach will be strictly valid only for the case that the *time-stretched* signal is band-limited to half the sampling frequency.

2.3 Fractal Reconstruction (FR)

It is important to note that the fractal reconstruction discussed in [18, 19] uses equally spaced samples. In [3] and this study, a time-stretching transformation similar to that used for the Shannon reconstruction has therefore been used prior to the fractal reconstruction. Note that the fractal reconstruction can be implemented in different ways. Strahle used target points (mid-points) to close the system of mapping equations. In the present study with non-equally spaced data, this method was very unstable and very sensitive to the smallest variation of velocity value, a property not desirable in light of the noise expected to accompany LDA measurements. Results presented by Chao and Leu[3] using target points appear promising, however it is not clear that their data set reflected the true particle arrival statistics of LDA, since they first artificially reduced the data rate, possibly preferentially, although unintentionally. Therefore in the present study, the fractal reconstruction was implemented using a fixed d_n coefficient in the defining relation

$$\begin{pmatrix} \tau_{new} \\ h_{new} \end{pmatrix} = \begin{pmatrix} a_n & 0 \\ c_n & d_n \end{pmatrix} \begin{pmatrix} \tau_{old} \\ h_{old} \end{pmatrix} + \begin{pmatrix} e_n \\ f_n \end{pmatrix}$$

where τ is the abscissa of the time-stretched coordinate system and h is the transformed velocity. The other transform coefficients were determined by demanding collocation on every interpolation interval. An obvious weak point of this reconstruction method for LDA data is that the essence of the technique, namely the similarity in scales, is not preserved after the reverse time transformation.

2.4 Projection onto Convex Sets (POCS)

Interpolation through **POCS** has been applied in a variety of fields in which a finite bandwidth B of the underlying process exists and is known. A function $g(t)$ which, through digital filtering (rectangular) satisfies this criteria, i.e. $G(\omega) = 0$ for $\omega > 2\pi B$, is then interatively matched to the measured velocities $u(t_i)$ using the iteration algorithm

$$g^{(k+1)}(t) = g^{(k)}(t) + \lambda_i \frac{u(t_i) - u_r(t_i)}{||\vec{h}(t_i)||} \vec{h}(t_i)$$

where λ_i is a relaxation factor,|| || is the L_2-norm and $u_r(t_i) = \vec{h}(t_i) g(t)$. Again there are several approaches to implementation, all of which are very computationally intensive. Principally however, the spectrum of the interpolation is band-limited according to the filter applied to $g(t)$. In the present study $g^o(t)$ was a low-pass filtered linear interplation of the LDA data, repressented by discrete values at interals of $1/2B$. The one-dimensional vector $\vec{h}(t_i)$ is defined for this implementation of $g(t)$ as:

$$[si(2\pi B(\tau_1 - t_i)), \ldots, si(2\pi B(\tau_M - t_i))]$$

where τ_i are equidistant sampled points of $g(t)$. Note that this method is not necessarily collocative, depending on the termination criteria of the interation.

Previously, this method has only been applied by Lee and Sung[12] to LDA data, with apparent success, extending the cut-off limits of the spectrum by 5-6 times over S+H. Unfortunately, they do not provide information on their choice of B and in fact, their good agreement with the target spectrum may well be fortuitous, the true deviation being compensated by aliasing. More details are required of their implementation technique to be conclusive.

2.5 Direct Spectral Estimation

For comparison purposes a direct spectral estimation has also been performed. This estimator is similar to that proposed by Roberts *et al.*[15] with an additional weighting of the individual velocity values $w(t_i)$, in this case with the residence time:

$$S(f) = T \frac{\left[\left| \sum u(t_i) w(t_i) d(t_i) e^{-2\pi j f t_i} \right|^2 - \sum u^2(t_i) w^2(t_i) d^2(t_i) \right]}{\left[\sum w(t_i) \right]^2}$$

where $d(t_i)$ is a window function. In [13] this estimator was shown to be superior to an estimator without residence time weighting, in terms of turbulence-induced velocity bias.

3 Description of Simulation and Experimental Data Sets

3.1 Test Signals

Two test signals were used to illustrate the reconstruction schemes in time domain, as shown in Fig 2. The first is an exponentially modulated cosinus of the form

$$y_1(t) = e^{\frac{t-15s}{12s}} \cos(\pi t / 1s)$$

The second is a simulated (1D) flow field with a mean of zero, a variance of 1 m^2/s^2 and an integral time scale of 1s. The particle density was chosen to yield a mean particle rate of 1 particle per integral time scale.

3.2 Simulation Data Sets

In addition to the two test signals previously mentioned, several other simulated data sets were investigated, all with a mean velocity of 10 m/s, an integral time scale of 0.1s, a first order autoregressive spectral distribution and a primary generation of 100 samples per time scale. The turbulence intensity was varied between 10% and 100% using both 1D and 3D flowfield simulations. All simulated data sets extended over 10,000 integral time scales. The particle number depended on the seeding density, which was varied to yield an average date density (particles per integral time scale) of $\alpha = 10$, 1 or 0.1. Residence times were generated for each particle passage using measurement volume dimensions of 20μm x 20μm x 100μm. Upon reconstruction, further processing continued only after a resampling at equidistant intervals of 0.01s. Full details of the simulation technique can be found in previous publications [8, 9].

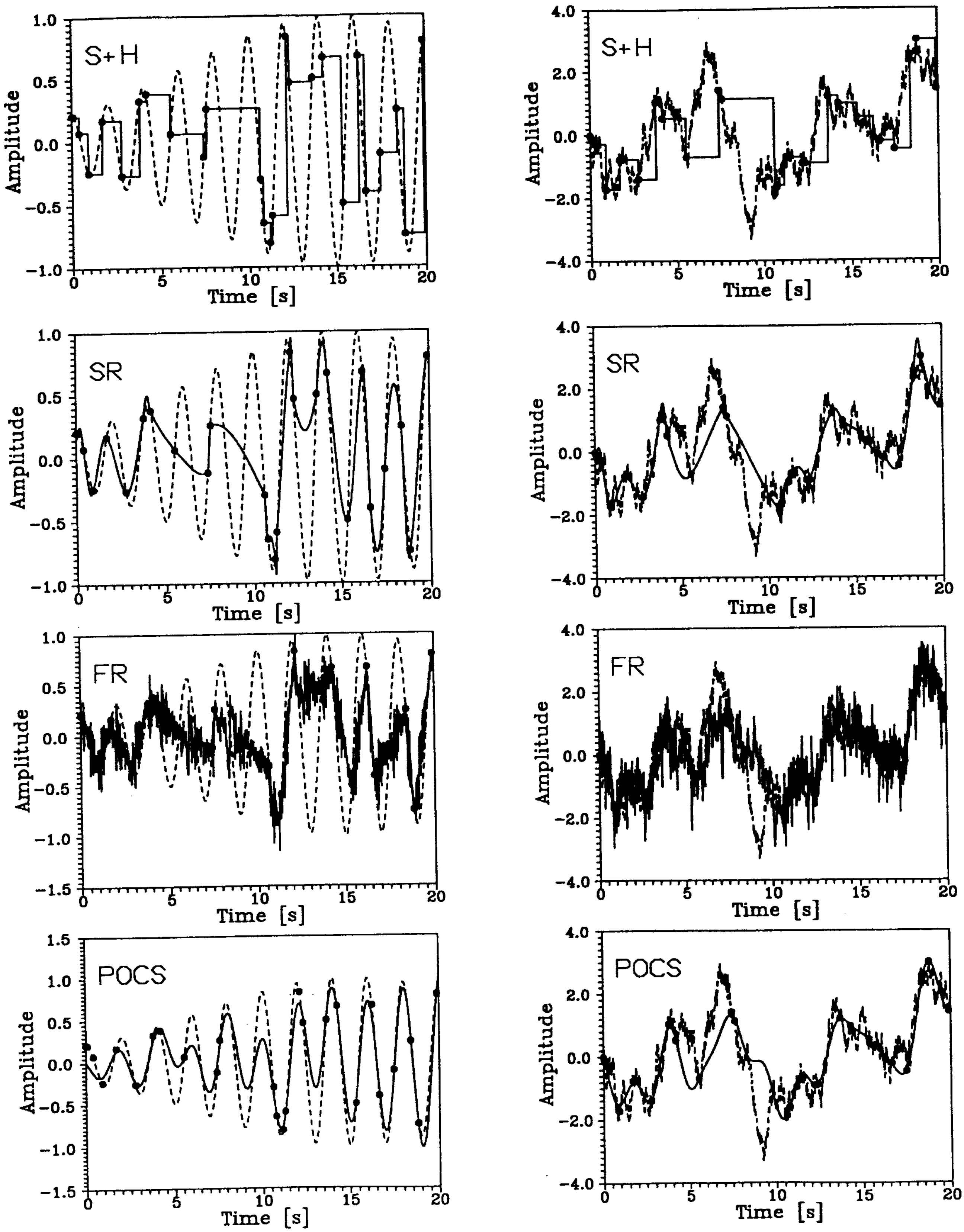

Figure 2: Reconstructed test signals using S+H, SR, FR and POCS interpolation: _ _ _ test signal; • LDA sample points; ——— reconstruction

3.3 Experimental Data Sets

Flow velocity measurements were performed behind a wall-mounted obstacle, allowing variation of the turbulence intensity. Hot-wire measurements and LDA measurements were performed at identical positions, the former without seeding. Only the measurement point 1 will be used in this study, the statistics of which are summarized in Table 1. As indicated in Fig.1, the HWA data was used also for indirect simulation studies, whereby the statistical values shown in Table 1 were used as a basis.

The primary purpose of including an experiment in this investigation was to verify the simulation procedure. This is illustrated in Fig. 3. In this figure the spectrum measured using the LDA, employing Shannon reconstruction is compared to the spectrum obtained directly from the hot-wire signal and with the simulated LDA signal using the hot-wire signal as a primary series in the simulation. This comparison shows that the simulation procedure yields a spectrum very similar to that which was actually measured with LDA, indicating that the numerical seeding procedure is trustworthy. Further reference to the hot-wire and LDA measurements will not be made.

HWA					LDA
ϑ_u[s]	$\bar{u}$[m/s]	σ_u^2[m^2/s^2]	D_o[-]	Tu[%]	$\dot{N}$[Hz]
0.003	8.156	0.141	1.67	4.6	212

Table 1: Statistics of measurement point 1

4 Results and Discussion

Already the interpolations viewed in time domain (Fig. 2) indicate that the success of any scheme will depend largely on the input signal. The band-limited test signal $y_n(t)$ is excellently approximated by **POCS**, since this

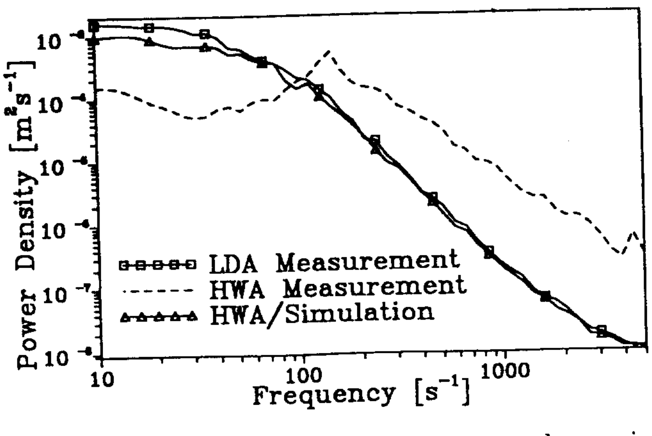

Figure 3: Verification of simulation procedure using HWA signal as primary series. A Shannon reconstruction was used for spectral estimation.

is a prerequisite for such an interpolation. Further evaluation of the reconstruction schemes however, can only be on the basis of more precise statistical measures. For this purpose the normalized error of the first two moments, using arithmetic averaging of the reconstructed signal, has been computed and presented in Fig. 4 for the **S+H**, **linear** and the **SR** reconstruction schemes. Also shown is the percent error expected for a free-running processor[7]. Note that the abscissa is now the data density, i.e. the data rate($\dot{N}$) times by the integral time scale(ϑ_u).

As expected, the normalized errors decrease with increasing data density. The first moment error of all estimators follows very closely the **S+H** error, which has also been theoretically derived and is shown in Fig. 4[23]. From these measures alone, no scheme can be strongly favoured over the other. Examining the normalized variance error indicates that the linear interpolation exhibits significantly lower variance estimates. The **SR** estimates lie between the **linear** and **S+H** values.

Spectral estimates based on a resampling of the reconstructed signal are shown in Fig. 5 for two data densities, $\alpha = 10$ and 0.1, corresponding to data rates of 100 ad 1 Hz respectively. They are compared to the expected spectrum, computed directly from the primary simulation series. All interpolation based estimators are expected to exhibit a low-pass filter characteristic, the cut-off frequency being related to the data density. This calls for great caution in appraising estimators, since typical turbulent spectra, and also the simulation models, resemble closely first order filters. For the **S+H** reconstruction, Adrian and Yao[1] have given the cut-off frequency as the data rate divided by 2π. This corresponds to 16 and 0.16 Hz for the two simulations respectively.

From Fig. 5 it is clear that the linear reconstruction and the Shannon reconstruction behaves more like a second order filter, falling off more rapidly than the **S+H** reconstruction. This also explains the lower variance observed in Fig. 4 for the linear reconstruction. The high variability of the direct estimation at high frequencies, as derived by Gastor and Roberts[10], is confirmed in Fig. 5. At low data densities all estimators resemble the **S+H** result, showing an increase of power at low frequencies, attributed to step noise in [1], and a decrease at higher frequencies due to the filter effect. In fact, the residence time weighted direct spectrum appears to be overall the most efective estimator from these data. Again it must be emphasized that the fact that the **S+H**, **Linear** and **SR** spectra fall off at higher frequencies have less to do with the fact that they estimate the flow fluctuations well, rather this is the *particle rate* filter effect. If the filter effect were computationally removed, these spectra would resemble the flatness of the direct estimate, but with significantly larger bias.

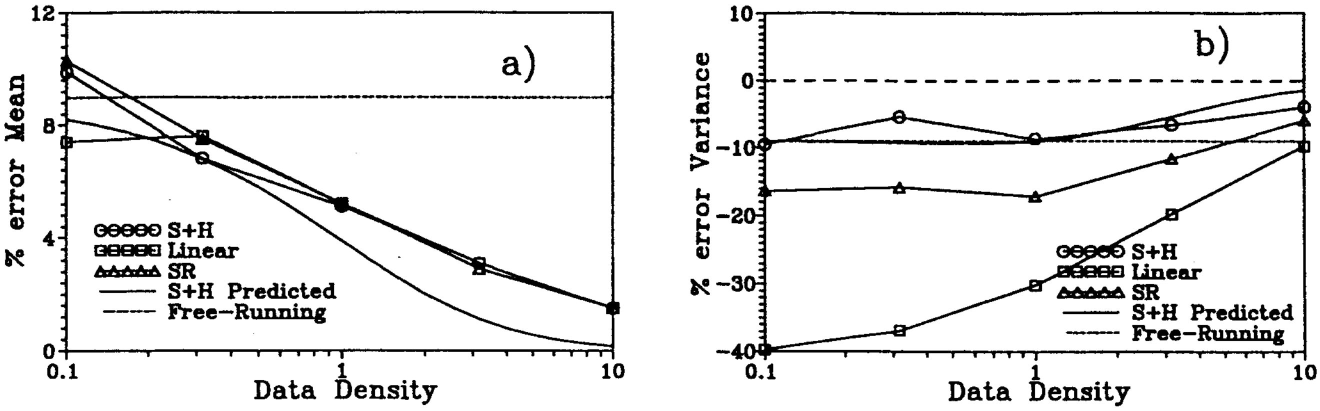

Figure 4: Percent error of a) mean and b) variance when arithmetically averaged over reconstructed signal. A 1D simulation with 30% turbulence level was used.

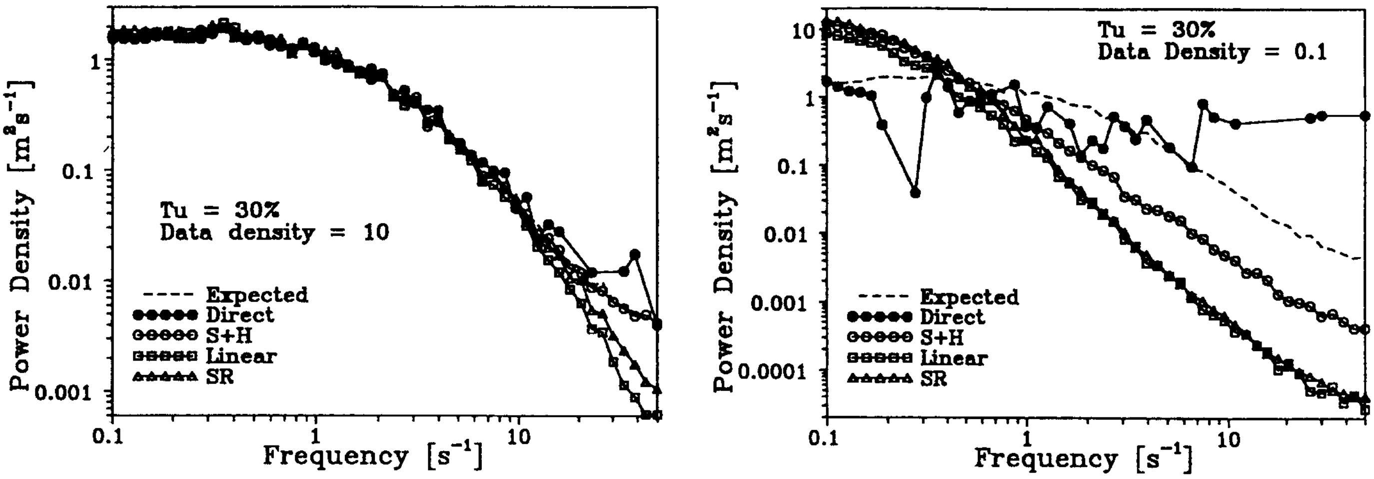

Figure 5: Spectral estimates from a 1D flow simulation with 30% turbulence and two data densities.

The estimate based on the Shannon reconstruction typically lies between the S+H and the linear reconstruction. In [21] this reconstruction scheme appeared to be superior to others, a behaviour which is surely attributable to the bandwidth limitation of the three sinusoids used as a test signal. In a second presentation of spectral estimates from Shannon reconstruction, no direct comparison to other techniques were explicitly given[22]. Note that one assumption in applying this technique is the bandwidth limitation of the time-stretched signal, a feature which by no means has been demonstrated for typical LDA signals. Originally this technique was applied only to regularily sampled signals with some jitter on the sample times, a considerably different situation to LDA data[4].

The difficulty in applying fractal reconstruction to LDA data is illustrated well in Fig. 6a, showing the dependence of the mean and variance bias on the choice of d_n. Although a value of 0.17 appears to be quite effective in minimizing bias errors of the variance, this is not known beforehand and furthermore the optimal value will also depend on turbulence level and data density. The mean bias does not reach zero even for a d_n value of 0.4.

Using the fixed value $d_n = 0.17$, the spectra for $\alpha = 10$ and 0.1 using the fractal reconstruction are shown in Fig. 6b. At high data densities the fractal reconstruction lies between the S+H and SR in the upper frequency range. At low data densities the FR estimate agrees well with the linear and SR estimates, i.e. it exhibits a stronger filter effect than the S+H estimate.

These results do not support the optimism of [3] in applying FR to LDA data. Possible refinements to improve the FR spectral estimation are: application of FR without the time-stretching transformation; estimation of the fractal dimension (D_o) from randomly sampled points and; formulation of some relation between D_o and d_n.

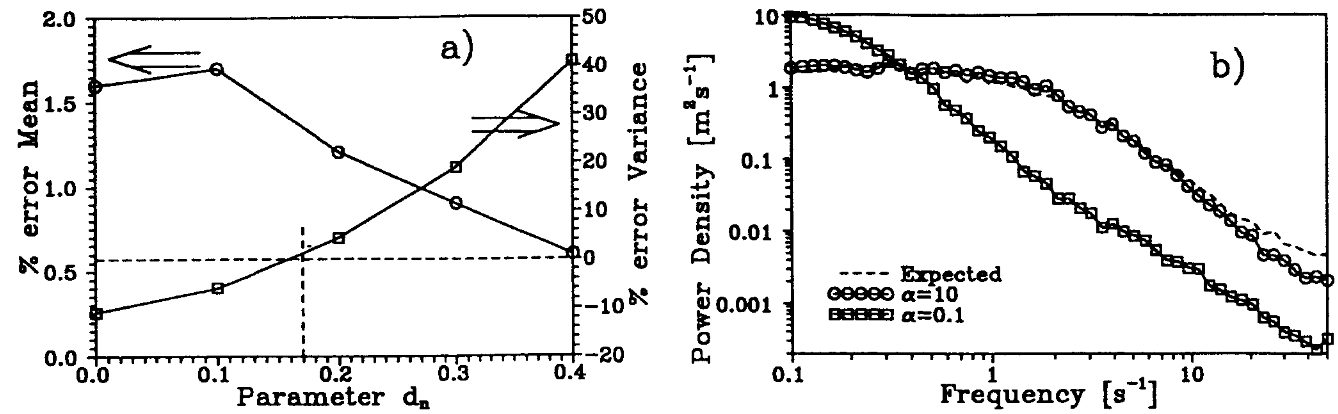

Figure 6: **a)** Influence of d_n on mean and variance bias of reconstructed signal; **b)** spectral estimates using fractal reconstruction with $d_n = 0.17$.

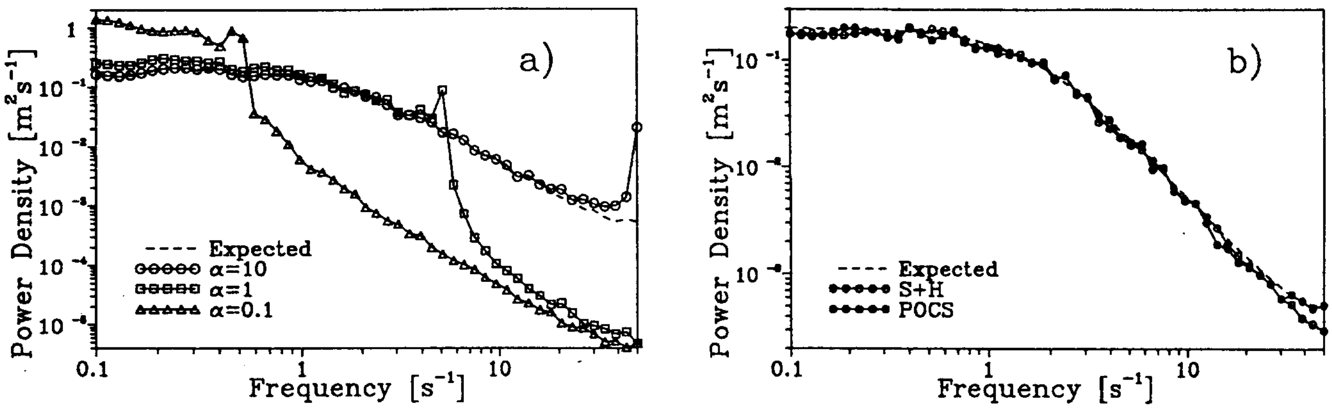

Figure 7: Spectral estimates using a POCS reconstruction: a) $B = \alpha/2$; b) $B = \alpha/1.6$, α=10.

Finally the results of applying the POCS interpolation are presented in Fig. 7. In Fig. 7a the effect of data density at a bandwidth of B=1/2α is demonstrated. Here the filter influence is much more severe than with other interpolation schemes, exhibiting in addition an oscillation corresponding to the applied boxcar filter (Gibb's phenomenon). If B is increased beyond $\alpha/2$, the spectrum estimate can closely approach the expected spectrum, as shown in Fig. 7b for α=10, however the correct choice of B is not known beforehand. Furthermore, if B is increased still further, the POCS spectrum will asymptotically approach the Linear estimate, because the linear interpolation was used as a start value for $g(t)$. These results indicate that the POCS estimation is not a promising alternative in the present implementation, at least without some scheme for choosing B.

5 Final Remarks

The possibility of improving parameter estimation for LDA data through the use of signal reconstruction techniques has been investigated. Several reconstruction techniques have been considered, some of which have been reported on previously in the literature, usually together with promising results. The results of the present study however, have shown that of those techniques considered, none can unconditionally be considered superior to a residence time weighted direct estimation.

The implementation of the time-stretching transformation in the fractal reconstruction is unsatifactory, since the similarity of scales is not preserved. A refinement in this regard could bring improvements, although some doubt exists whether previously reported performance can be achieved with actual LDA data.

While the present results cannot clearly recommend a signal reconstruction as an alternative to the direct estimation, other conclusions may be obtained for the case of non-homogeneously seeded flows, as was examined in [21]. This will be the topic of future work.

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