Model Parameter Estimation from LDA Data at Low Particle Densities

6th International Conference on Laser Anemometry

August 13–18, 1995, Hilton Head Island, South Carolina, USA

E. Müller, H. Nobach
Universität Rostock
Institut für Nachrichtentechnik und Informationselektronik

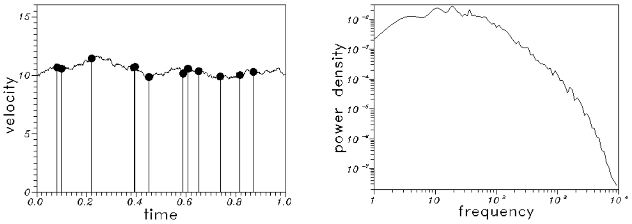
C. Tropea
Universität Erlangen
Lehrstuhl für Strömungsmechanik

ABSTRACT

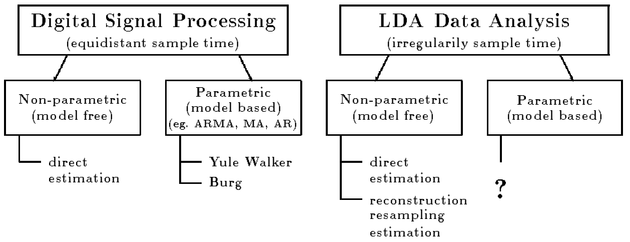
The estimation of spectra from LDA data using model parameter estimation is examined. An ARMA (Autoregressive Moving Average) process is used to model the flow velocity fluctuations and the model parameters are estimated through the autocorrelation function. This new technique is described in detail and its performance, particularily for low data densities is examined using simulations and experiments. The estimator is shown to be well suited, not only for low data densities, but also for short record lengths, as would be expected in some transient flowfields.

INTRODUCTION

The randomness of particle arrivals in the measurement volume of a laser Doppler anemometer (LDA) must be considered when computing statistical quantities of the velocity field. This is especially true for the estimation of the power spectral density, or spectrum, since the arrival times of particles will directly influence the frequency of velocity fluctuations which can be resolved.



A considerable number of spectral estimators for LDA data have been suggested in the past, several of which will be described below. Generally speaking however, most estimators perform well if the data density, i.e. the mean number of particles per integral flow time scale, is sufficiently high. The challenge to perform well is greater if the data density decreases and even more so, if at the same time, the data set is of short duration, for example in the case of transient flowfields such as in engines. This is precisely the situation which motivated the present work, which introduces a new LDA spectral estimator based on model parameter estimation.



Digital Signal Processing
(equidistant sample time)

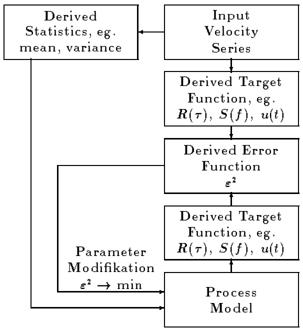
- Non-parametric (model free)
  - direct estimation
- Parametric (model based) (eg. ARMA, MA, AR)
  - Yule Walker
  - Burg

LDA Data Analysis
(irregularily sample time)

- Non-parametric (model free)
  - direct estimation
  - reconstruction resampling estimation
- Parametric (model based)
  - ?

MODEL BASED SPECTRAL ESTIMATION

The input velocity information is compared to the process model on the basis of a target function, such as the autocorrelation function or the spectrum. The deviations are evaluated as an error function, which is used to alter the process model parameters to iteratively achieve minimum deviation. The resultant parameter set then represents the best match of the model to the physical process.



Input Velocity Series
Derived Statistics, eg. mean, variance
Derived Target Function, eg. $R(\tau)$, $S(f)$, $u(t)$
Derived Error Function ε^2
Derived Target Function, eg. $R(\tau)$, $S(f)$, $u(t)$
Parameter Modifikation $\varepsilon^2 \rightarrow$ min
Process Model

Target function:

$$g_k = \sum_{i;j=1}^{N} u(t_i)u(t_j)$$

with $(k-1)\Delta\tau < t_j - t_i \leq k\Delta\tau$ and $i \neq j$

Error function:

$$\varepsilon^2 = \sum_{k=1}^{N_s} (R_k N_k - g_k)^2$$

with number of slots N_s, slot duration $\Delta\tau$, k-th value of ACF R_k, number of products N_k for g_k and number of samples N

PROCESS MODEL

Basis for matching the flow velocity fluctuations: **ARMA** (Autoregressive Moving Average) Process

- General form for time sequence [Box&Jenkins, 1976]

  $$z_k = \phi_1 z_{k-1} + \ldots + \phi_p z_{k-p} + a_k - \theta_1 a_{k-1} - \ldots - \theta_q a_{k-q}$$

  with order of AR process p, order of MA Process q and white noise a.
- for AR Process

  Time sequence

  $$z_k = \phi_1 z_{k-1} + \ldots + \phi_p z_{k-p} + a_k \qquad \phi_1, \ldots \phi_p \text{ - weight parameters}$$

  Autocorrelation function

  $$R_{zz}(k) = R_k = \phi_1 R_{k-1} + \phi_2 R_{k-2} + \ldots + \phi_p R_{k-p} \qquad (k > p)$$

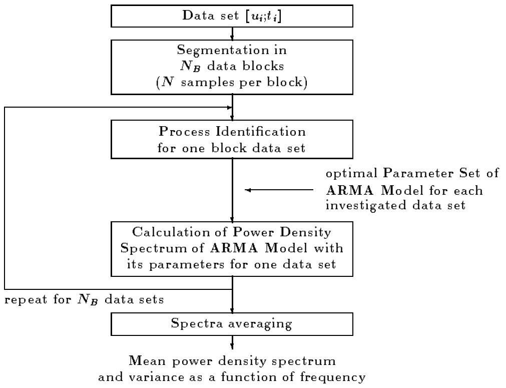
  Variance

  $$\sigma_z^2 = \frac{\sigma_a^2}{1 - \rho_1\phi_1 - \rho_2\phi_2 - \ldots - \rho_p\phi_p} \qquad \text{with } \rho_i = \frac{R_i}{R_0}$$

  Power density spectrum ("Spectrum")

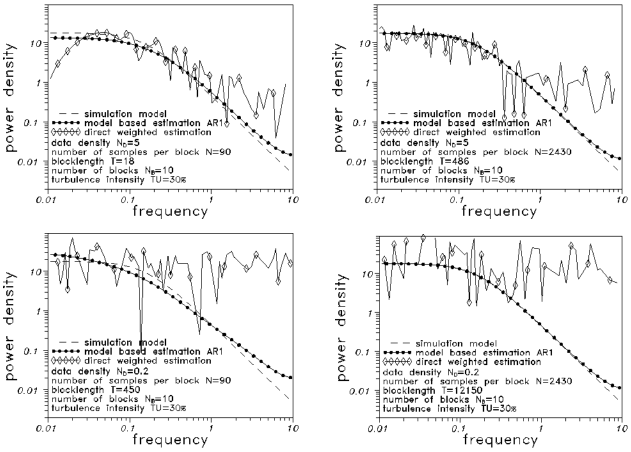
  $$S_{zz}(f) = \frac{2\sigma_a^2}{|1 - \phi_1 e^{-i2\pi f} - \phi_2 e^{-i4\pi f} - \ldots - \phi_p e^{-i2p\pi f}|^2} \qquad f \geq 0$$

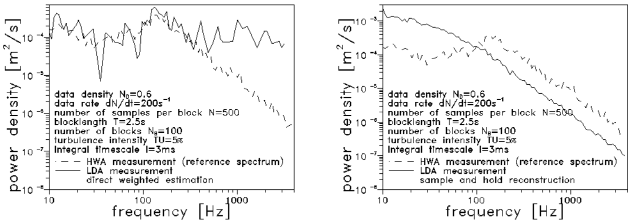
GLOBAL SPECTRAL ESTIMATION STRATEGY

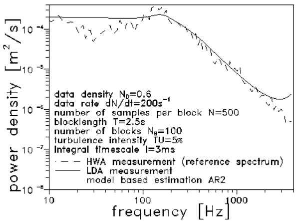


Data set $[u_i;t_i]$
Segmentation in N_B data blocks (N samples per block)
Process Identification for one block data set
optimal Parameter Set of **ARMA** Model for each investigated data set
Calculation of Power Density Spectrum of **ARMA** Model with its parameters for one data set
repeat for N_B data sets
Spectra averaging
Mean power density spectrum and variance as a function of frequency

RESULTS







HWA and LDA measurements performed behind a wall mounted obstacle. Data taken at 5% turbulence and data density of 0.6.

⇒ Model based spectral estimate is more efficient as direct estimate, especialy st low data densities.

⇒ Model based spectral estimate is clearly superior to the direct estimation in the higher frequency range.

CONCLUSIONS

- The new LDA spectral estimator presupposes that the physical process of velocity fluctuations can be described by a model (such as autoregressive model).
- The present model based spectral estimator yields especially reliable results for low data densities and short data records. It appears to work reliable also for very low data densities. The performance is clearly superior to well-known non-parametric LDA spectral estimation methods.
- At properties of the physical process which are not described by the choosen model, the present estimator doesn't perform well (eg. at dominant low frequencies). Nevertheless, the performance is not worse than other available estimators.
- Improvement may be achievable using other models or/and the combination of model based estimation and signal reconstruction.

Die vorgestellten Ergebnisse sind Resultate des durch die DFG geförderten Projektes Mu 1117/1-1.