MODEL PARAMETER ESTIMATION FROM LDA DATA AT LOW PARTICLE DENSITIES

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ABSTRACT

The estimation of spectra from LDA data using model parameter estimation is examined. An ARMA (Autoregressive Moving Average) process is used to model the flow velocity fluctuations and the model parameters are estimated through the autocorrelation function. This new technique is described in detail and its performance, particularily for low data densities is examined using simulations and experiments. The estimator is shown to be well suited, not only for low data densities, but also for short record lengths, as would be expected in some transient flowfields.

INTRODUCTION

The randomness of particle arrivals in the measurement volume of a laser Doppler anemometer (LDA) must be considered when computing statistical quantities of the velocity field. This is especially true for the estimation of the power spectral density, or spectrum, since the arrival times of particles will directly influence the frequency of velocity fluctuations which can be resolved. A considerable number of spectral estimators for LDA data have been suggested in the past, several of which will be described below. Generally speaking however, most estimators perform well if the data density, i.e. the mean number of particles per integral flow time scale, is sufficiently high. The challenge to perform well is greater if the data density decreases and even more so, if at the same time, the data set is of short duration, for example in the case of transient flowfields such as in engines. This is precisely the situation which motivated the present work, which introduces a new LDA spectral estimator based on model parameter estimation.

The large number of possible spectral estimators can be interpreted according to the broad classification shown in Fig. 1. The non-parametric methods generally assume no pre-knowledge of the velocity signal, and a prescribed computational procedure is followed, independent of the input data. Model based approaches assume a particular model or family of models for the signal or one of its derivative quantities, and the model parameters are adjusted to achieve a minimum deviation from the target function. With very few exceptions (Veynante and Candel, 1988a, van Maanen, 1994), previously proposed estimators have been non-parametric, presumably to avoid the necessity of having *a priori* knowledge about the signal.

Of the non-parametric approaches, both the direct estimation and signal reconstruction have been widely used and, for most estimators, have also been well analysed with respect to performance. The direct estimation can be performed using the discrete Fourier transformation (DFT), as proposed initially by Gastor and Roberts (1975) and discussed in more detail in subsequent publications (Gastor and Roberts, 1977, Roberts et al., 1980). Alternatively, the correlation function can first be estimated, eg. using a slot correlation, followed by a Fourier transform (Bell, 1986, Mayo, 1975, Scott 1974). The direct estimation as described in Roberts et al. (1980), will also be used in the present study for comparison purposes. The method has been slightly modified in that the individual velocity samples have been weighted with their residence time, as outlined in Müller et al. (1994). This has shown to yield a better estimate of the total power, especially at low data densities (Nobach, 1993).

Signal reconstruction with resampling at equidistant time intervals, as a means of spectral estimation, has received increased attention in recent years, although the zero order reconstruction – sample and hold (**S+H**) – dates back to the digital sampling of the analog output of tracker processors. In the meantime, the **S+H** reconstruction has been well analysed in terms of spectral content (Adrian and Yao, 1987) and in terms of moments (Edwards and Jensen, 1983). More complex reconstruction schemes have been proposed, including higher order polynomials, projection onto convex sets (POCS) (Lee and Sung, 1992), fractal reconstruction (Chao and Leu, 1992) and the so-called Shannon reconstruction (Veynante and Candel, 1988b, Clark et al., 1985). A recent examination of different schemes has led however, to the conclusion that none of these techniques can be considered universally superior to either the direct estimation or the **S+H** reconstruction (Müller et al., 1994).

Perhaps the most promising reconstruction approach to data has been presented as a *single exponential interpolation* (Høst-Madsen, 1994), in which the signal between samples can be described as

$$X'(t) = e^{-b(t-t^-)}X(t^-) \tag{1}$$

where t^- is the arrival time of the previous sample and b is a decay constant to be prescribed. A value of b=0 leads to the **S+H** reconstruction. Letting b tend to infinity, while at the same time scaling with b, represents the direct estimation. Excellent results were obtained using for the value of b the inverse of the integral time scale and thus, a decaying function between samples. Evaluation of the estimator was based on the *interpolation error*, defined as the added standard deviation of the spectrum. On this basis, the single exponential interpolation achieves at low data densities comparable results to the direct estimation and, at high data densities, results comparable to the **S+H** and is thus shown to be an appropriate estimator for the entire range. Further details of this technique can be found in Høst-Madsen (1994).

Despite the very good performance of the single exponential interpolation, it is reasonable to assume that improvements can be achieved if *a priori* information concerning the signal is available and can be entered into the estimation process. The parametric estimator introduced in the following section allows this. Its performance will however, clearly depend on how good the *a priori* assumptions match the physical process.

DESCRIPTION OF THE ESTIMATION PROCEDURE

Process Model

The estimation procedure is based on matching the measured velocity information to an ARMA (Autoregressive Moving Average) process, as described in its most general form for a time sequence as (Box and Jenkins, 1976):

$$z_k = \phi_1 z_{k-1} + \ldots + \phi_p z_{k-p} + a_k - \theta_1 a_{k-1} - \ldots - \theta_q a_{k-q} \tag{2}$$

where p and q are the orders of the AR and MA processes and a is the added white noise. In the following study, we will restrict ourselves to the AR process

$$z_k = \phi_1 z_{k-1} + \ldots + \phi_p z_{k-p} + a_k \tag{3}$$

with the finite set of weight parameters $\phi_1, \ldots, \phi_p$. The variance of the process is given by

$$\sigma_z^2 = \frac{\sigma_a^2}{1 - \rho_1\phi_1 - \rho_2\phi_2 - \ldots - \rho_p\phi_p} \tag{4}$$

with

$$\rho_i = \frac{R_i}{R_0} \tag{5}$$

using the autocorrelation function

$$R_k = \phi_1 R_{k-1} + \phi_2 R_{k-2} + \ldots + \phi_p R_{k-p} \qquad k > p \tag{6}$$

The spectrum of the AR process is given by

$$S(f) = \frac{2\sigma_a^2}{|1 - \phi_1 e^{-i2\pi f} - \ldots - \phi_p e^{-i2\pi p f}|^2} \tag{7}$$

The integral time scale of the process is defined as

$$I = \int_0^\infty |\rho(\tau)| d\tau \tag{8}$$

Error Function

The basic procedure to be followed is shown schematically in Fig. 2. The input velocity information is compared to the process model on the basis of a target function, such as the autocorrelation function or the spectrum. The deviations are evaluated as an error function, which is used to alter the process model parameters to iteratively achieve minimum deviation. The resultant parameter set then represents the best match of the model to the physical process. Further iterations can be performed to optimize the model order. Also initial statistics of the velocity information, such as mean or variance, can be used as preset and fixed values for the process model.

In the present case, a function related to the autocorrelation function has been used as a target function. This function is designated as g_k. For the velocity series, this function is computed using a slotting technique, with N_s slots, each of duration $\Delta\tau$, typically 1-5% of the model integral time scale. In computing the target function g_k however, the sum of the cross products accumulated in each slot is *not* divided by the number of products used.

$$g_k = \sum_{i,j=1}^{N-1} u(t_i)u(t_j) \qquad (k-1)\Delta\tau < t_j - t_i \leq k\Delta\tau \tag{9}$$

Furthermore, in the first slot, no self-products are included. The error function is then computed as

$$\epsilon^2 = \sum_{k=1}^{N_s} (R_k N_k - g_k)^2 \tag{10}$$

where N_k is the number of products in Eq. (9) contributing to the function g_k. Thus, the square error in slots with more products contribute more to the total error, resulting in a linear weighting of ϵ^2 with the number of products in the slot. Note that this is not equivalent to forming the error function

$$\hat{\epsilon}^2 = \sum_{k=1}^{N_s} (R_k - g_k/N_k)^2 \tag{11}$$

since this estimate will be strongly biased when N_k is small, as is mostly the case. Eq. (10) also illustrates why the self products are neglected in the first slot. Otherwise they would contribute disproportionately to the error.

Iteration Algorithm

The goal is to minimize ϵ^2 by choosing an appropriate model parameter set. This is straightforward for a model of first order, in which only the parameter ϕ_1 must be found. For a second order model, local minima can appear, depending on the step width used for iteration. Therefore a special search strategy has been developed, applicable for all model orders. The method will be illustrated for order 2.

The search begins by computing ϵ^2 for the initial weight parameters $\phi_1 = \phi_2 = 0$ (white noise). Using an initial step size of $\Delta\phi_1 = \Delta\phi_2 = 0.5$, the change of ϵ^2 is computed for the eight vector directions (1,0), (1,1), ... (1,-1) as indicated in Fig. 3. The direction with the largest decrease in ϵ^2 is chosen and the search continues from this point with the same step size, again searching in eight directions. This continues until a minimum in ϵ^2 is found, upon which the step size is halved. The search is terminated when the step size reaches 10^{-12}. Note that the weight parameters are valid only for the chosen slot width $\Delta\tau$.

Fig. 4 illustrates a few selected iso-curves of ϵ^2 in the $\phi_1 - \phi_2$ plane, computed using a very small step size grid. The correct weight parameters are $\phi_1 = 0.7$ and $\phi_2 = 0.2$. At an intermediate iteration, a local minimum is seen to lie a considerable distance away from the correct value. Note that this search procedure can be extended easily to higher orders. The number of local search direction vectors increases correspondingly.

Spectral Estimate

The iteration algorithm is carried out for each block of data, yielding the optimized weight parameters for that block. This parameter set can then be used to form a continuous spectrum according to Eq. (7). The final spectral estimate is taken as the average spectrum of N_b blocks of data. Thus, this estimator yields a continuous spectrum over the frequency, but does not explicitly provide a reconstruction of the time series.

The estimation can be continued by repeating the procedure using a higher order model. The absolute block error will necessarily decrease with increasing order, meaning that the data set will be increasingly better represented by the model. However, the improvement is marginal at higher orders, at a stage where signal noise is also being matched by the model. Criteria for choosing an optimum order can be very subjective. For the present situation, the *minimum information theoretic criterion* (MAICE) introduced by Akaike (1974) has been used when required.

Time Series Reconstruction

A time series reconstruction is possible using the autocorrelation function, Eq. (6).

$$u_{i+k\Delta\tau} = u_i \rho_k \tag{12}$$

Two cases are examined more closely. For a first order process, the interpolation becomes

$$u_{i+k\Delta\tau} = u_i \phi_1^k \tag{13}$$

Thus, the interpolation is an exponential decay from the last velocity sample, with a decay parameter (1-ϕ_1). Since, however, the integral time constant of a first order AR process is just $\Delta\tau/(1-\phi_1)$, this is equivalent to the single exponential interpolation presented by Høst-Madsen (1994). For a second order process, a velocity value is required at a time $\Delta\tau$ before the last LDA sample. Since this value is not available in the data set, it must be generated in some manner. One possibility would be to invoke the symmetry of the autocorrelation function about the sample point.

SIMULATION AND EXPERIMENTAL DETAILS

The performance of the new spectral estimator has been examined using both simulations and experiments. The simulation is based on the very extensively developed simulation techniques reported by Fuchs et al. (1992) and Fuchs et al. (1994). These techniques allow LDA data sets to be generated on the basis of a simulated, three-dimensional velocity series with a prescribed data density. The underlying process is an autoregressive process of desired order, generated for very small time steps, typically 100 times per integral time scale. The use of simulated data sets has the advantage that the true flow statistics are known.

Alternatively, experiments can be performed with the LDA. However, it is desirable in this case to also measure the flow velocity with a hot-wire anemometer (HWA), to have comparison data available. In the present case, HWA and LDA measurements were performed in the wake of a square cylinder.

Finally, a combination of experiment and simulation is possible and has proven very valuable in the past. The hot-wire signal, sampled at a high data rate (> 100/integral time scale), can be used as an input primary series to the simulation, rather than the AR process mentioned above. Particle arrivals can then be generated on this series, representing an LDA data set but having exactly the statistics of the HWA signal. In this way the LDA estimator can be compared to the HWA without actually performing an LDA experiment. This technique has been shown to be very reliable, by comparing such simulations with real LDA measurements (Fuchs et al., 1994). The combination technique has the advantage of not being restricted to an AR process for the simulation.

RESULTS

Simulations

The simulation is first used to investigate the influence of data density and block size on the spectral estimation. The flowfield has been simulated with a turbulence intensity of 30%, allowing a one-dimensional, instead of a three-dimensional flowfield simulation (Fuchs et al. 1992). A higher level of turbulence would lead to higher turbulence-induced bias errors of the spectrum, as discussed in Tropea (1987). However, in the present study, the variance and not the bias of the estimator is of more interest initially. A first order autoregressive process with an integral time scale of 1 time unit was used. The data density was varied between 0.2 and 5, a range which is considered low for moment estimation.

The results are summarized in Fig. 5 for three data densities and three block sizes. The function g_k has been computed using $N_k = 64$, with a slot width of $\Delta\tau = 1/20$ time units, thus the spectrum will be available up to a frequency of 10 (1/time unit). The block size has been varied between $N = 10$ and $N = 2430$ LDA samples, representing a block length of approximately $T = 2$ to 12150 time units, depending on the data density. All results have been obtained by averaging the spectra of 10 blocks.

In each diagram of Fig. 5, four curves have been included: the theoretical AR spectrum, the model based spectral estimate (MB), the direct spectrum with residence time weighting (D) and the sample and hold (S+H) reconstruction. In the case of the direct spectrum, the reciprocal of the velocity was used as a weight rather than the residence time, since the flow was one-dimensional. Thus, the residence time simulation stage could be avoided.

The model based estimate is seen to be, in general, an improvement over both the direct and the **S+H** estimates. At high frequencies, an aliasing effect in the model based result is evident. This could be avoided by decreasing the slot width $\Delta\tau$. However, then N_s must be increased above 64 to maintain the same frequency range. This increases the computational load considerably.

At small block sizes, all methods show large deviations from the true spectrum. Note however, that only 100 data points (20 integral time scales) are being used in total for these estimates (10×10). The direct method also shows artifacts of the rectangular window for low frequencies.

For the low data densities ($N_D = 0.2$), the 'step noise' and 'particle rate filter', as discussed by Adrian and Yao (1987) are apparent for the **S+H** estimate. The model based estimate is not influenced in this manner and continues to show good agreement with the true spectrum, at least for $N \geq 270$. In this simulation example therefore, the model based estimated is particularly advantageous for low data densities where, to date, no suitable alternative has been found.

The results of Fig. 5 illustrate also that the model based spectrum is very smooth, in contrast to the irregular curves of the direct estimate and the **S+H** estimate. This is somewhat misleading, since indeed the model based spectrum has a variance. This can be estimated as the variance over the 10 spectra which contribute to the presented result. This variance will not change with the number of blocks (only the variance of the averaged estimate will!). However, it does change with the block duration, either through the data density or the number of data samples per block. This is illustrated in Fig. 6, where the model based spectra for selected conditions in Fig. 5 have been replotted, adding error bars to indicate the variance over the 10 blocks averaged. As the block duration increases from $T = 54$ to $T = 270$ and then to $T = 2430$, the variance decreases. At low frequencies, the variance is larger, since fewer 'periods' at those frequencies have been acquired and processed.

A second simulation has been performed to indicate the effect of the order of the model chosen. In this case, an LDA data set has been generated using a second order AR process with an integral time scale of 1 time unit and a data denstiy of 5. A spectral estimate based on one block of 1000 LDA samples was computed for both a first and second order model. The results of this simulation are presented in Fig 7. Clearly, the first order model is inappropriate and the second order model performs very well, as expected.

Experiments

One experiment was performed in which both LDA and HWA measurements were carried out. Details of this experiment can be found in Müller et al. (1994). The turbulence intensity was 4.6%, with an integral time scale of 3 ms. The average particle rate was 212 s^{-1}, giving a data density of $N_D = 1.6$.

The results of this experiment are illustrated in Fig. 8. In each of the two diagrams, the spectrum of the hot-wire signal is used as a reference. The model based (MB) estimate, performed for a second order AR model, is clearly superior to the direct estimation in the higher frequency range. The periodicity in the signal is, however, not as well distinguished by the model based estimate. This presumably is due to the low data density and the high estimator variance at these frequencies. At higher frequencies, at which the HWA signal indicates a second slope in the spectrum, the model based estimate also fails. This is a consequence of the fact that a second order AR process is simply not a good description of the physical process, i.e. the *a priori* information used is not appropriate.

Simulations Based on Experiments

A second experiment was performed to generate HWA data, suitable to be used as a primary series for further simulation. Measurements were performed eight diameters downstream of an axisymmetric air jet, on the centerline. The turbulence level was approximately 19%, the mean velocity 9.55 m/s and the integral time scale 3 ms. The HWA signal was sampled at 20 kHz (60 times per integral time scale). The spectrum of this signal was used as a reference, with which to compare the LDA estimate.

LDA data was then generated at data densities of N_D = 0.5, 1.6 and 5.0, using the HWA data and the simulation program. The results of first order, model based spectral estimation are shown in Fig. 9, together with direct estimation and **S+H** estimation. In each case, 20 blocks, each 300 LDA samples long, were processed. The data rate is also shown in each diagram for reference.

The HWA signal clearly shows two distinct slopes in the spectrum, as was the case in the previous experiments presented in Fig. 8. None of the estimators are successful in capturing this feature of the spectrum, regardless of the data rate. In the case of the **S+H** spectrum, the 'filter' effect takes over at approximately $\dot{N}/2\pi$, i.e. in the best case at approximately 200 Hz, which lies below the frequency of the second slope ($\approx$ 1200 Hz). At 'high' data densities ($N_D = 5$), the **S+H** and model based estimates are similarly good, whereas the direct estimate is unsatisfactory. However, for the data densities of $N_D = 1.6$ and $N_D = 0.5$, the model based estimate is clearly advantageous. The **S+H** estimated begins to exhibit 'step noise' and the effect of the 'particle filter'. An estimation of the spectrum using a second order AR process did not lead to any significant improvements.

CONCLUDING REMARKS

A new LDA spectral estimator has been introduced, which presupposes that the physical process of velocity fluctuations can be described by a model, in this case an autoregressive model. As expected, in situations or at frequencies at which this assumption is not valid, the new estimator does not perform well. However, it is apparently not worse than other available estimators. The model based estimator, on the other hand, appears to yield especially reliable results for very low data densities, much superior to other alternative LDA spectral estimators. Improvements could be expected if a model were available which, like turbulence, exhibited two distinct slopes in the spectrum. The authors are not aware of such an appropriate model as yet.

Alternatively, hybrid approaches may be interesting to investigate, for instance, the combination of model based estimation and signal reconstruction. Certainly, a direct comparison of the single exponential interpolation technique with the model based technique would be enlightening in this respect and will be the subject of future investigations.

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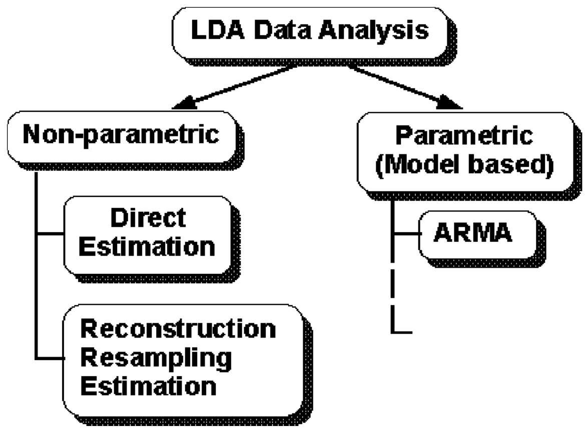

Figure 1: Overview of estimation possibilities for LDA data

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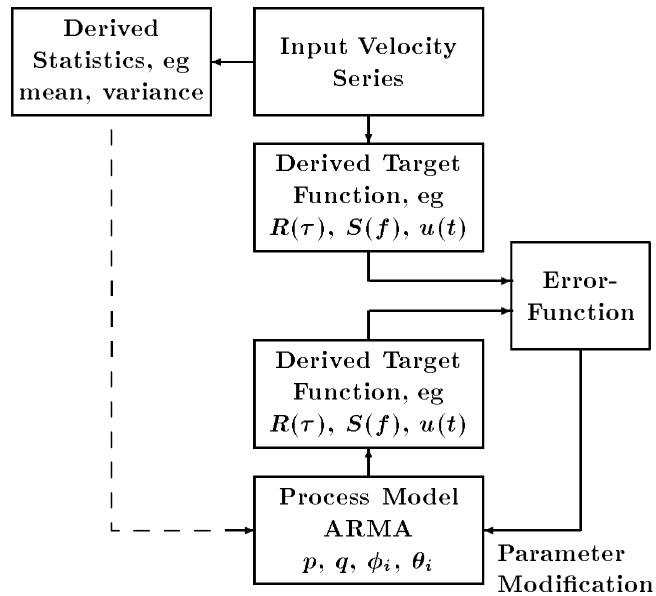

Figure 2: Procedure for model parameter estimation

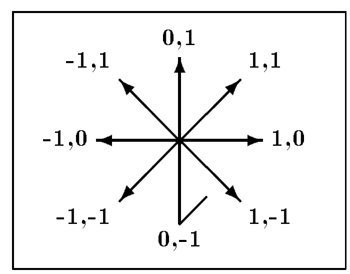

Figure 3: Graphic representation of search vectors for second order AR process

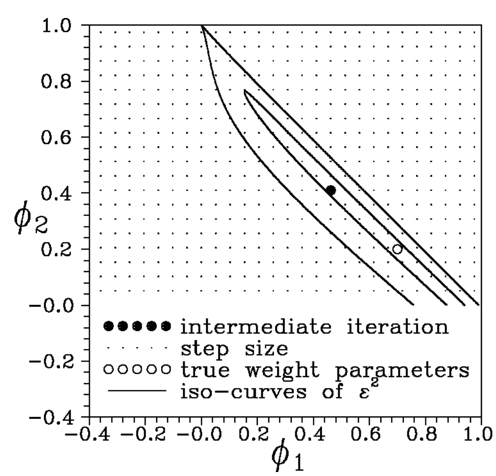

Figure 4: Iso-curves of error function in ϕ_1 - ϕ_2 plane

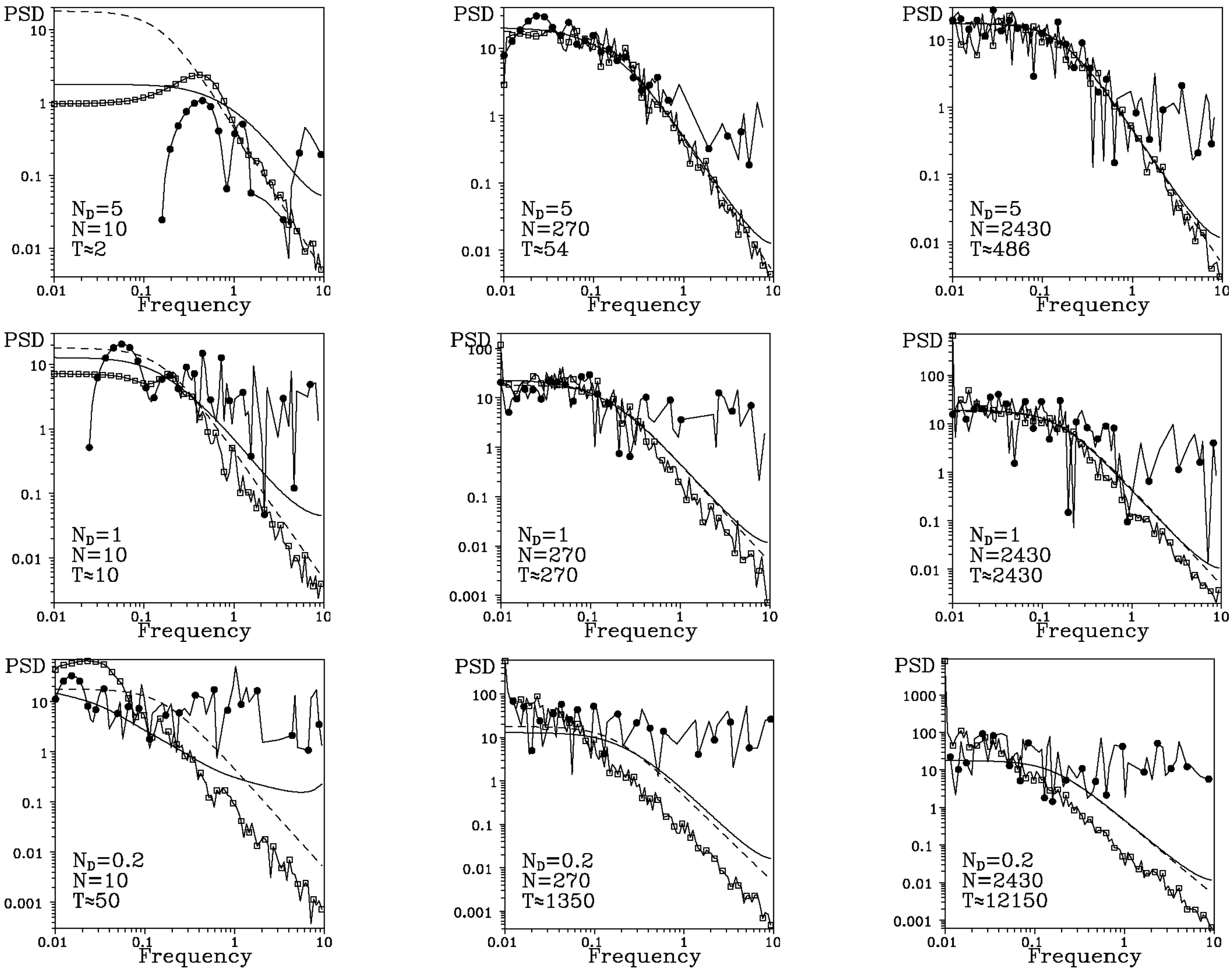

Figure 5: Results using a first order AR process simulation (Tu = 30%, 10 block averages): – – – simulation model, ——— Model based estimation, □ **S+H** estimate, • direct estimation, N_D data density, N number of samples, T blocklength

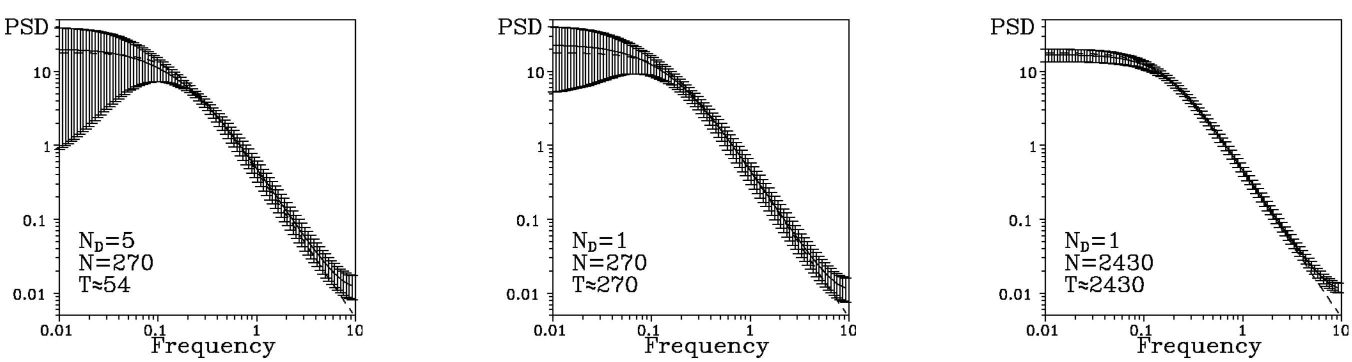

Figure 6: The variance of the estimated spectra over 10 blocks of data

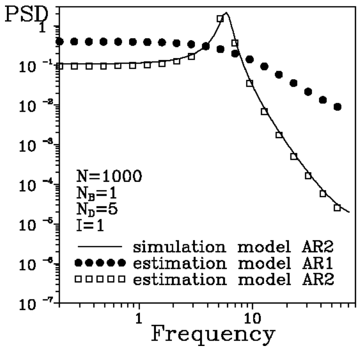

Figure 7: Estimation of an AR(2) process using a first and second order model

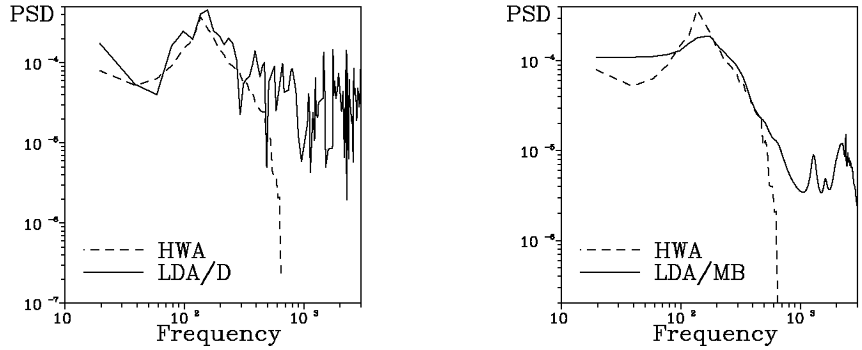

Figure 8: Comparison of HWA and LDA spectrum: a) using direct estimation, b) using model based estimation

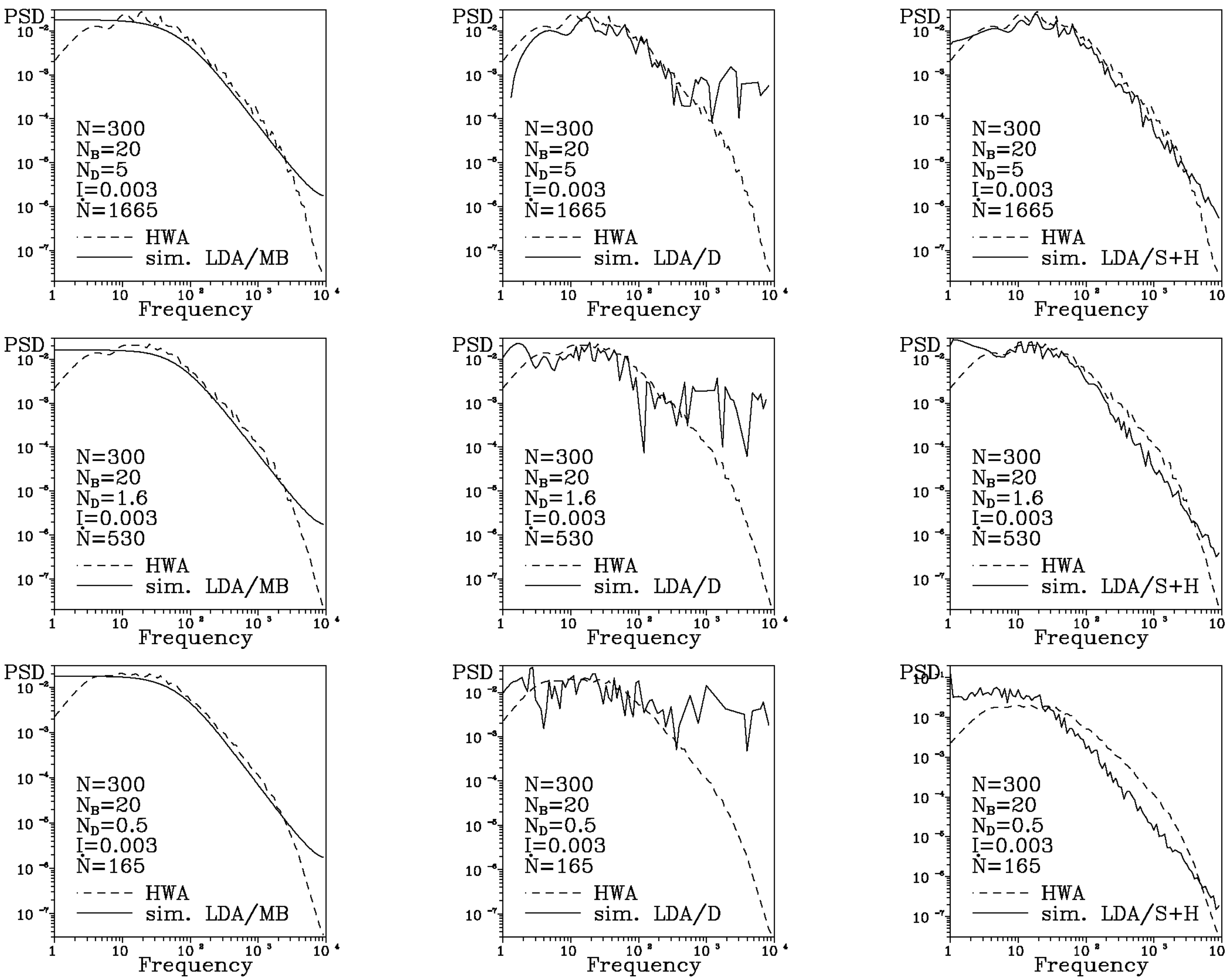

Figure 9: Comparison of model based (MB), direct (D) and sample and hold (S+H) spectral estimators for a data set generated from a measured HWA signal