# LDA Benchmark Generator III

Holger Nobach

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### Abstract

A program is described, which generates simulated, irregularly sampled LDA data sets suitable for testing processing algorithms or for comparing various estimation procedures of statistical quantities. It builds on a previous version designed to help evaluate spectral estimation algorithms and has been improved. The program is available on the internet at the address http://www.nambis.de/benchmark.

### 1 Introduction

The test of data processing algorithms for laser Doppler anemometry (LDA) requires data sets with known statistical characteristics. Except for some trivial cases, this is seldom possible via experiments and thus simulation routines can be particularly valuable. However, the correct simulation of LDA data is not trivial due to many subtle physical effects relating to both, the optical sampling of the flow velocity and the particular signal processing employed [1]. The experience of the last 9 years and especially the results of recent benchmark tests for the estimation of power spectra from LDA data [2] has led to a revised generation program with several new features, like different effects of the velocity bias, processor delay, periodic components in the velocity fluctuations, varying data rates and drop outs. In its present form the program has been released to generate LDA data sets of 5 pre-defined spectral types with 4 different types of bias simulation and 64 groups with different characteristics (data rate, noise, processor delay, velocity oscillations, varying data rates and drop outs). Each set of any type and group is referenced with an index number. In this way users on different machines will be able to reproduce exactly the same data sets. The data sets themselves are otherwise far too large to transfer directly. Each data set comprises the arrival time and the velocity value of each sample in ASCII format. The program bm3 is available freely via internet at http://www.nambis.de/benchmark.

## 2 LDA Data Model

The basic simulation concept can be found in [1]. The equidistant sampled primary velocity time series (figure 1) is sampled non-regularly by the simulation of particle arrivals. The final step is the simulation of the processor effects, like time delay and noise. The simulation routines are described in detail in the next sections.



Figure 1: Signal flow of the LDA generation process.

Parameter	Symbol	Value
Primary Sampling Rate	$f_{ m p}$	$20.0\mathrm{kHz}$
Filter Order	q	249 (MA) for Spectral Type 1
		299 (MA) for Spectral Type 2
		2499 (MA) for Spectral Type 3
		2499 (MA) for Spectral Type 4
		2 (AR) for Spectral Type 5
Velocity Variance	$\sigma_{\rm u}^2$	$1.0  { m m}^2 / { m s}^2$
Oscillation Amplitude	Ā	$3.0\mathrm{m/s}$
Oscillation Frequency	F	$0.1\mathrm{Hz}$
Oscillation Phase	$\Phi$	random
Mean Velocity	m	$2.0\mathrm{m/s}$
Cross Velocity Component	v	$1.0\mathrm{m/s}$

Table 1: Simulation parameters for the primary velocity series.

#### $\mathbf{2.1}$ **Primary Velocity Time Series**

In the first simulation step a primary velocity series is generated with given spectral characteristics. There are five different spectral types as illustrated in figure 2.

- 1. Band-limited random noise with Gaussian amplitude distribution and a very steep roll-off at 300 Hz.
- 2. A Pao-like spectrum, which decreases exponentially (in log coordinates) with increasing frequency

$$S(f) \sim 2^{-f/300\,\text{Hz}}$$
 (1)

3. A Heisenberg spectrum with

$$S(f) \sim \frac{1}{\left(1 + (f/100 \,\mathrm{Hz})^{5/3}\right) \left(1 + (f/1000 \,\mathrm{Hz})^{16/3}\right)} \tag{2}$$

4. The Heisenberg spectrum with two distinct peaks at 100 Hz and 1000 Hz

### 5. An AR2 spectrum with a strong resonance

The primary velocity series is equidistantly spaced with the sampling frequency of  $f_{\rm p}$ . The sectral types 1–4 are generated using a moving average (MA) filter of order q (figure 3)

$$u'_{\mathrm{p}i} = \sum_{j=0}^{q} \theta_j e_{\mathrm{p}(i-j)} \tag{3}$$

from an uncorrelated noise  $e_p$  with a Gaussian distribution. The filter coefficients  $\theta_j$  are chosen in such a way that the resulting series  $u'_{pi}$  has a given power spectral density (PSD) S(f) and a given variance  $\sigma_{u}^{2}$ . The spectral type 5 is generated using an autoregressive (AR) filter of 2<sup>nd</sup> order.

$$u'_{pi} = e_{pi} + 1.98u'_{p(i-1)} - 0.985u'_{p(i-2)}$$
(4)

leading to a strong resonance.

In an additional step the final primary velocity series is found through

$$u_{\mathrm{p}i} = u'_{\mathrm{p}i} + m + A\cos(2\pi F t_{\mathrm{p}i} + \Phi) \tag{5}$$

with the mean velocity m and a periodic component with the amplitude A, the frequency F, the phase  $\Phi$ and the sample times  $t_{pi}$ . Examples of three different simulated primary time series are shown in figure 4.

The specifications of all quantities are given in table 1.



Figure 2: Spectral types: a) Band-limited random noise; b) Pao-like spectrum; c) Heisenberg spectrum; d) Heisenberg spectrum with two peaks; e) Autoregressive spectrum.



Figure 3: Generation filter for the primary velocity time series.



Figure 4: Primary velocity time series: a) mean free; b) with a mean velocity and c) with an oscillating mean velocity.



Figure 5: Secondary velocity time series: a) without any distortion; b) with simulated velocity bias at a high turbulence level; c) with a varying data rate and d) with drop outs.

### 2.2 Particle (Secondary) Time Series

The irregular sampling of the velocity time series is simulated through a Poisson process, which calculates a random number of velocity samples within a given simulation step of the primary velocity series.

$$P(N) = \frac{\alpha^N}{N!} e^{-\alpha}$$
(6)

The coefficient  $\alpha$  depends on the simulated mean data rate with several modifications (varying data rate, drop outs) and, if the velocity bias is considered, also on the velocity including a mean velocity or a cross component.

Through different relations between the mean velocity m in the measurement direction and the cross component v it is possible to generate data sets with different velocity bias conditions, e.g. a positive bias of variance estimation without a mean or cross velocity, a negative bias with a strong mean velocity and a non-proportional relationship between the data rate and the measured velocity component through the use of a cross component.

In any case, the arrival times of the particles will not coincide with the sampling times of the primary velocity series. To obtain the velocity at the particle arrival times a zeroth order interpolation of the primary series is used, i.e. the value of the previous sample is used, as illustrated in figure 6. The resulting continuous velocity function has a new PSD. It differs from the PSD of the primary series, e.g. it has no aliasing errors. Figure 7 shows the spectra of the three types after the sample-and-hold interpolation.

The specifications of all quantities are given in table 2.

### 2.3 Processor (Tertiary) Time Series

The processor simulation assumes that only a single frequency measurement per burst is performed. The transit time of each particle is calculated through a random process simulating the particle trajectory through the measurement volume considering also the particle velocity. The dimensions of the measurement volume



Figure 6: Interpolation of the primary time series at the computed particle arrival times.



Figure 7: Spectral types after sample-and-hold interpolation: a) Band-limited random noise; b) Pao-like spectrum; c) Heisenberg spectrum; d) Heisenberg spectrum with two peaks e) Autoregressive spectrum.

Parameter	Symbol	Value
Variable Data Rate		
Rate Ratio		4:1
Periode		$1.0\mathrm{s}$
Drop Outs		
Time Ratio (data:no data)		4:1
Periode		$3.1\mathrm{s}$
Particle Rate		$100 \mathrm{s}^{-1}$ (low data rate)
		$1000 \mathrm{s}^{-1}$ (high data rate)

Table 2: Simulation parameters for the secondary velocity series.

Parameter	Symbol	Value
Noise Power	$\sigma_{ m n}^2$	$0.2 { m m}^2/{ m s}^2$
Processor delay	$t_{\rm d}$	$1.0\mathrm{ms}$

Table 3: Simulation parameters for the tertiary velocity series.

are derived so that a particle, crossing the measurement volume parallel to the measurement direction with the mean velocity and in the center of the measurement volume, leads to a transit time which is equal to the given process delay  $t_d$ .

Simulated velocity samples are accepted only if the transit times of the appropriate particles do not overlap. In a final step a white noise with a Gaussian distribution and a power of  $\sigma_n^2$  is added to the velocity values, corresponding to the uncertainty of the frequency estimation in the signal processor.

The specifications of all quantities are given in table 3.

### 3 Using the Programs and Program Parameters

The generation program bm3 is available as a DOS-executable (bm3dos.exe) and as a Windows (32 bit) executable (bm3win.exe). It can be started through

#### bm3.exe name spectraltype biastype group seriesindex

### or

### bm3.exe name spectraltype biastype group series index samples

where bm3.exe is either bm3dos.exe or bm3win.exe, name is the name of the output file without an extension, spectraltype (1...5) specifies the spectral type (see section 2.1, biastype (0...3) specifies whether the velocity bias is simulated or not and the the bias character (see table 4), group (0...63) defines the data characteristics (see table 5) and seriesindex (0...) is the index of the individual realisation. The index 0 leads to random individual realisations, whereas indices  $\geq 1$  lead to defined data sets. Using different values for seriesindex leads to independent data sets, the same value leads to the same data set. This can be used to generate the same data sets on different machines to process them by different estimation routines. The parameter samples is optional. It defines the number of samples to be generated (default 100000).

The program generates the output file name.ts containing the LDA data set as an ASCII text. Each row gives one sample of the format

Arrival Time	Velocity	Transit Time
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where the values are given in the floating point representation

[-] < digit>. < digits> E[+|-] < exponent>

and the values are separated by single spaces.

Beside this, the program generates a file name.coe containing the coefficients of the filter for the generation of the primary velocity series. The additional program co3.exe calculates the theoretical spectrum and the autocorrelation function of the interpolated primary time series (see figure 7). It can be started with

#### co3.exe name f1 f2 Nf

Bias Type	Effect	<b>Bias Simulation</b>	Mean Velocity	Cross Component
0				
1	+	×		
2	—	×	×	
3	*	×		×

 $\times$  yes

+ positive variance bias (high turbulence level)

- negative variance bias (low turbulence level)

\* data rate $\not\sim$  measured velocity (cross component)

### Table 4: Specification of the bias types.

where name is the name of the coefficients file without an extension, f1 and f2 (> 0) give the frequency range of the spectrum and Nf is the number of spectral values. Because the theoretical spectrum of the interpolated velocity function is not sampled, it is possible to calculate the PSD for any frequency. The program co2.exe divides the spectrum equidistant on a logarithmic scale within the given frequency range. The result is written as ASCII data to the file name.spc. Each row gives one value of the spectrum

Frequency Power Spectral Density

where the values are given in the floating point representation. Furthermore, the program co3.exe calculates the theoretical autocorrelation function of the primary time series. The result is written as ASCII data to the file name.acf. Each row gives one value of the function

### Time Lag Correlation

where the values are given in the floating point representation. This function is sampled equidistantly with the sampling frequency  $f_{\rm p}$  of the primary velocity series. Nevertheless, the theoretical correlation function is continuous. It can be reconstructed from the sampled function by a linear interpolation (the linear interpolation is exact for the used series generator for the spectral types 1–4. For a time lag  $\geq q f_{\rm p}$  with the order q of the filter, the theoretical correlation is zero.

With the group parameter the individual characteristics of the LDA data set can be chosen. Table 5 gives an overview of all possible groups.

### References

- W Fuchs, H Nobach, and C Tropea. Laser Doppler anemometry data simulation: Application to investigate the accuracy of statistical estimators. AIAA Journal, 32:1883–1889, 1994.
- [2] L H Benedict, H Nobach, and C Tropea. Benchmark tests for the estimation of power spectra from LDA signals. In Proc. 9th Int. Symp. on Appl. of Laser Techn. to Fluid Mechanics, Lisbon, Portugal, 1998. paper 32.6.

Group	Drop Outs	Varying Data Rate	Oscillation	Processor Delay	Noise	Data Rate	
0						Ŷ	
1						↓ ↑	
2					×		
3					×	↓ ↑	
4				×			
5				$\sim$	$\sim$	↓ ↑	
7				$\hat{\mathbf{x}}$	$\hat{\mathbf{x}}$		
8			×	~	~	↓ ↑	
9			×			l	
10			×		×	Ť	
11			×		×	Ļ	
12			×	×		↑	
13			×	×		Ļ	
14			$\times$	×	$\times$	Ŷ	
15			×	×	×	$\downarrow$	
16		×				Î	
17		×				$\downarrow$	
18		×			×	Î	
19		×			$\times$	$\downarrow$	
20		×		×		Î	
21		×		×		Ļ	
22		×		×	×	Î	
23		×		×	×	↓ ¢	
24		×	×			T	
25 96		×	×			↓	
20 97		X	×		×		
21 28		×	×	~	×	↓ ↑	
$\frac{20}{20}$		$\sim$	$\mathbf{\hat{v}}$	$\sim$			
30		$\mathbf{\hat{x}}$	$\hat{\mathbf{x}}$	$\hat{\mathbf{x}}$	×	↓ ↑	
31		×	×	×	×		
-						~	

Group	Drop Outs	Varying Data Rate	Oscillation	Processor Delay	Noise	Data Rate
32	×					Î
33	×					$\downarrow$
34	×				×	$\uparrow$
35	×				×	$\downarrow$
36	×			×		$\uparrow$
37	×			×		$\downarrow$
38	×			×	×	$\uparrow$
39	×			×	×	$\downarrow$
40	×		$\times$			$\uparrow$
41	×		$\times$			$\downarrow$
42	×		$\times$		×	Î
43	×		$\times$		×	$\downarrow$
44	×		$\times$	×		↑ (
45	×		$\times$	×		$\downarrow$
46	×		$\times$	×	×	↑ (
47	×		$\times$	×	×	$\downarrow$
48	×	×				↑ (
49	×	×				$\downarrow$
50	×	×			×	↑ (
51	×	×			×	$\downarrow$
52	×	×		×		Î
53	×	×		×		$\downarrow$
54	×	×		×	$\times$	Î
55	×	×		×	$\times$	$\downarrow$
56	×	×	$\times$			Î
57	×	×	$\times$			$\downarrow$
58	×	×	$\times$		$\times$	Î
59	×	×	$\times$		$\times$	$\downarrow$
60	×	×	$\times$	×		Î
61	×	×	$\times$	×		Ļ
62	×	×	$\times$	×	×	Î
63	×	×	$\times$	×	$\times$	$\downarrow$

yes high low  $\stackrel{\times}{\uparrow}$ 

Table 5: Specification of the group characteristics.