

# Laser-Doppler-Korrelationsmessungen in turbulenten Strömungen

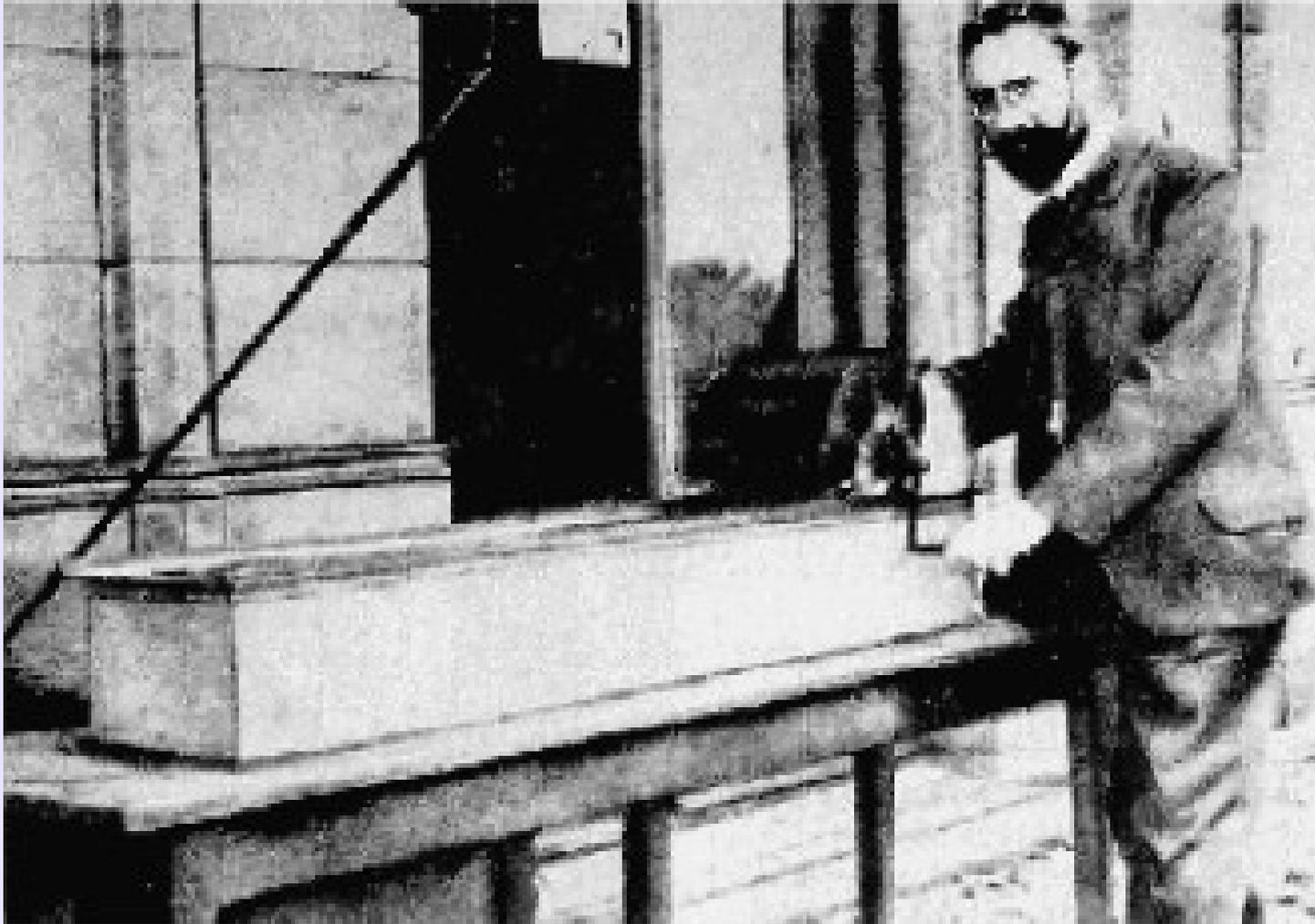
Holger Nobach

Max-Planck-Institut für Dynamik und Selbstorganisation  
Göttingen

TU Dresden, 17.11.2006



# Max-Planck-Institut für Dynamik und Selbstorganisation



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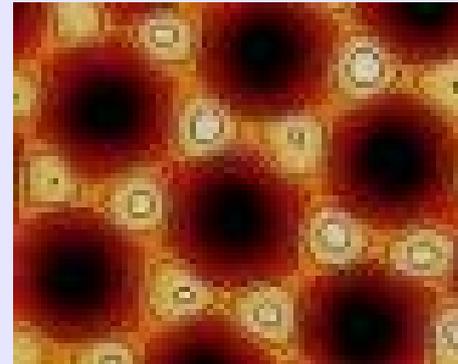
Nichtlineare Dynamik



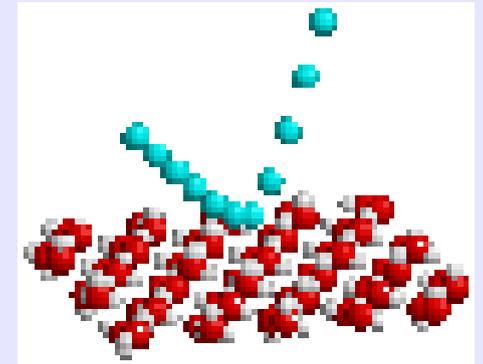
Dynamik komplexer Fluide



Hydrodynamik,  
Strukturbildung und  
Nanobiokomplexität

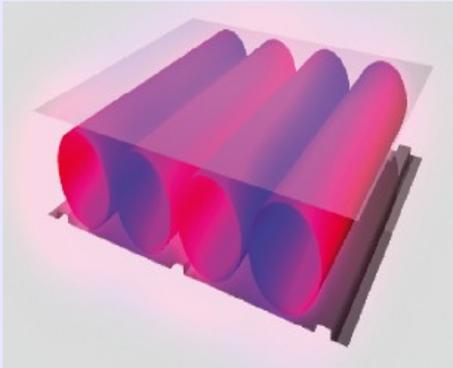


Molekulare  
Wechselwirkungen

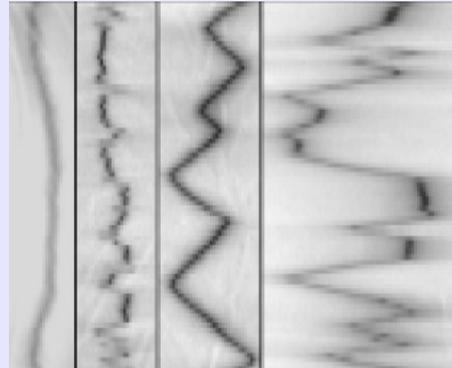


# Abteilung Hydrodynamik, Strukturbildung und Nanobiokomplexität

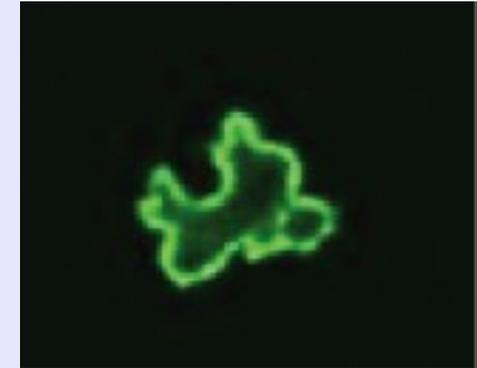
Thermische Konvektion



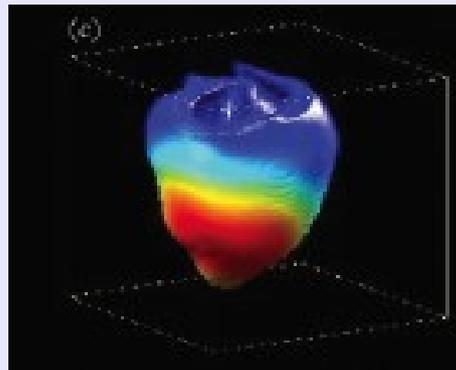
Tektonik



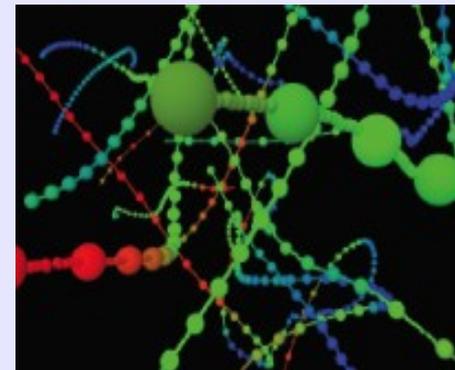
Chemotaxis



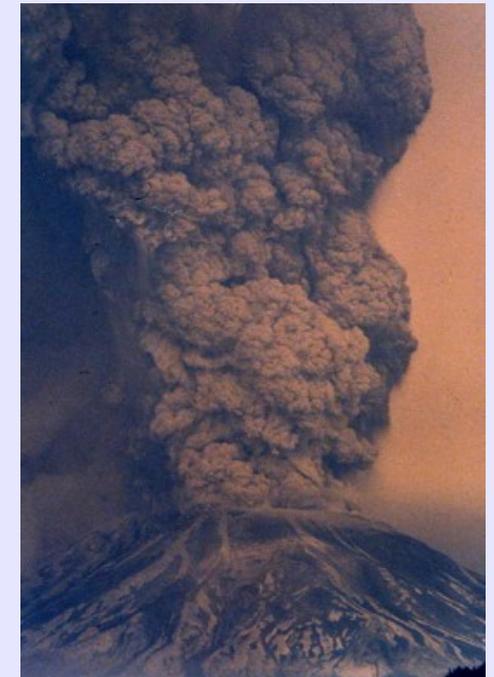
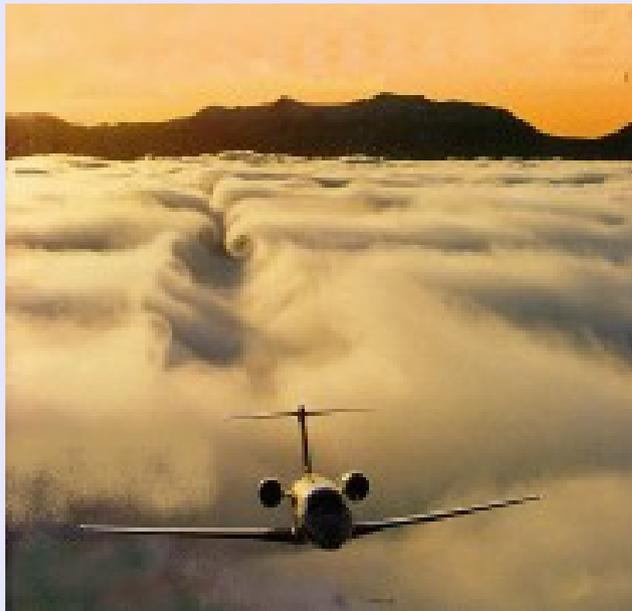
Herzdynamik



Turbulenz



# Turbulenz



# Navier-Stokes-Gleichung

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \eta \Delta \mathbf{u} + (\lambda + \eta) \nabla (\nabla \cdot \mathbf{u}) + \mathbf{f}$$

Randbedingungen  $\Leftrightarrow$  Direkte Numerische Simulation (DNS)

+ alle Strömungsparameter zugänglich



# Navier-Stokes-Gleichung

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \eta \Delta \mathbf{u} + (\lambda + \eta) \nabla (\nabla \cdot \mathbf{u}) + \mathbf{f}$$

Randbedingungen  $\Leftrightarrow$  Direkte Numerische Simulation (DNS)

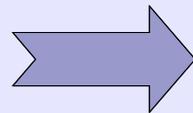
- + alle Strömungsparameter zugänglich
- numerisch aufwendig
- einfache (periodische) Randbedingungen
- keine realen (komplexen) Strömungen

# Navier-Stokes-Gleichung

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \eta \Delta \mathbf{u} + (\lambda + \eta) \nabla (\nabla \cdot \mathbf{u}) + \mathbf{f}$$

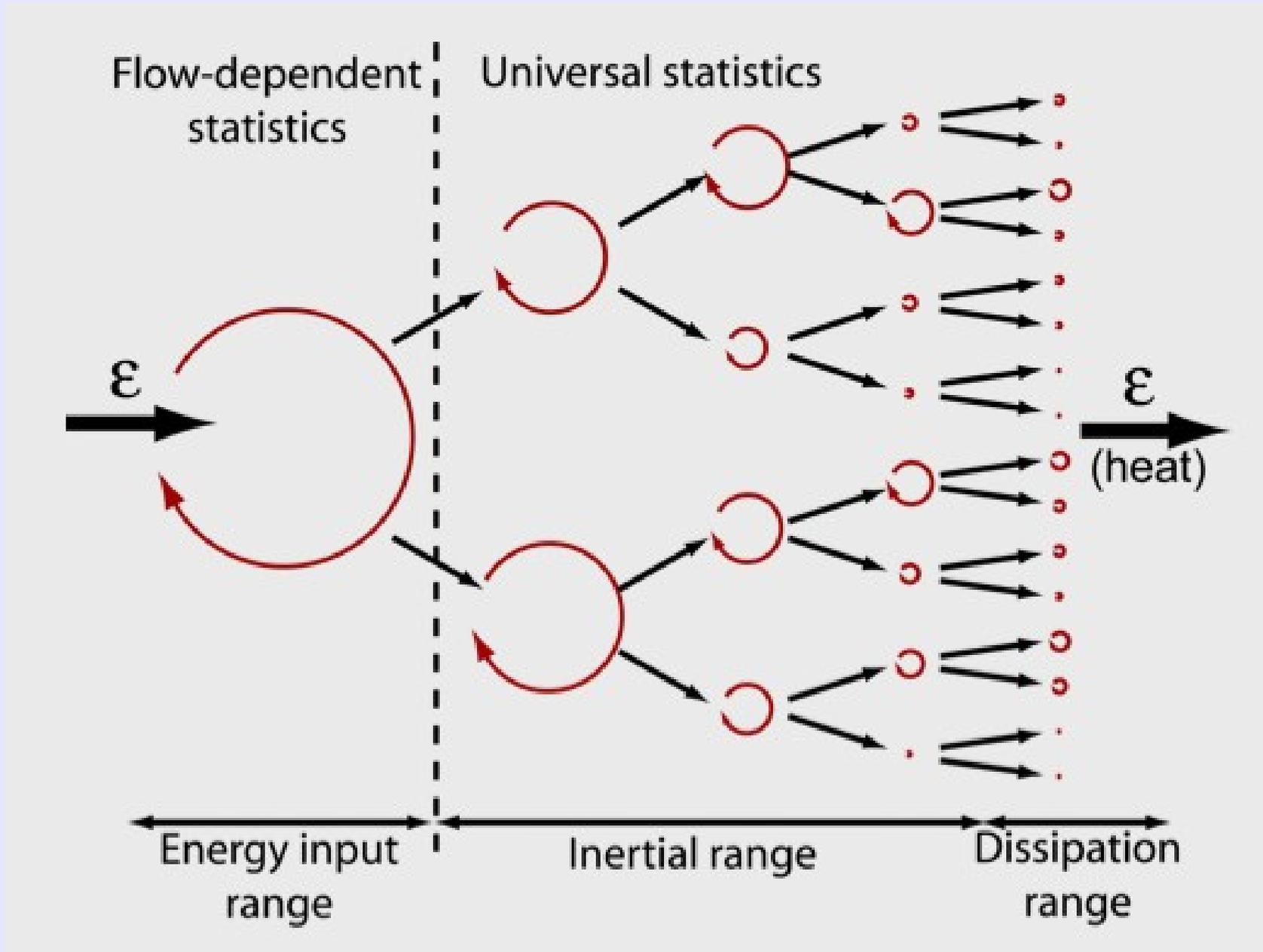
Randbedingungen  $\Leftrightarrow$  Direkte Numerische Simulation (DNS)

- + alle Strömungsparameter zugänglich
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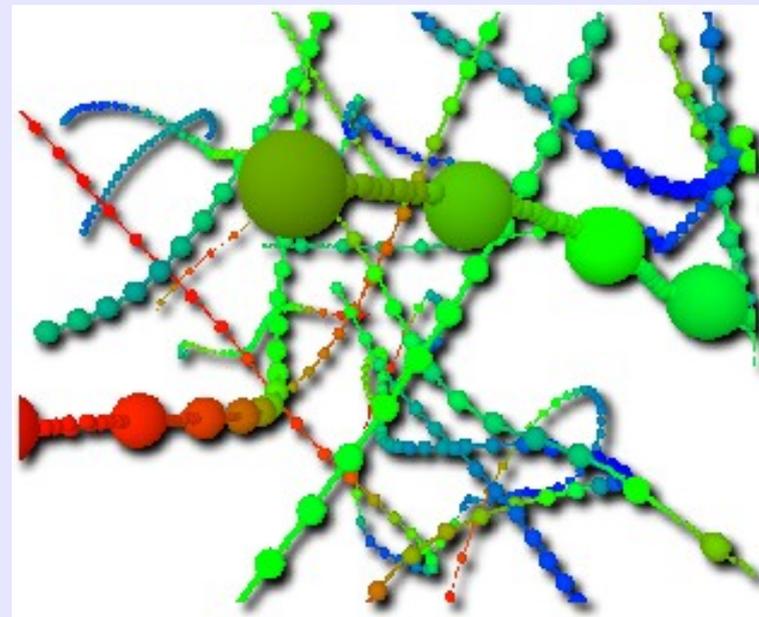
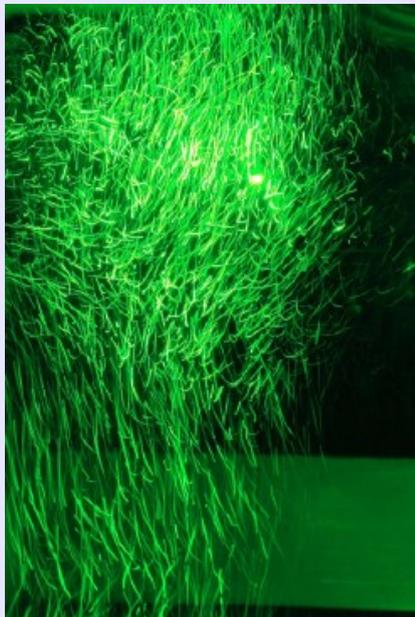
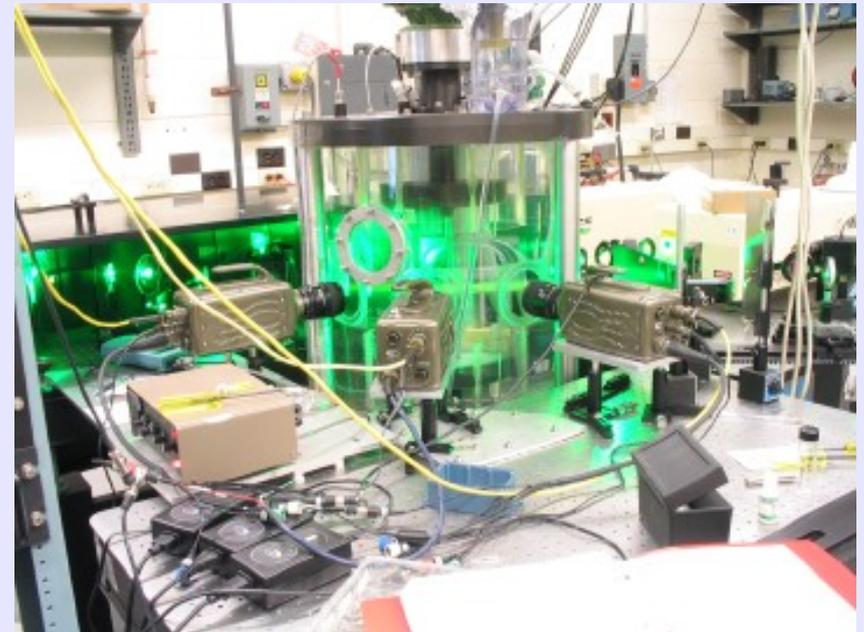
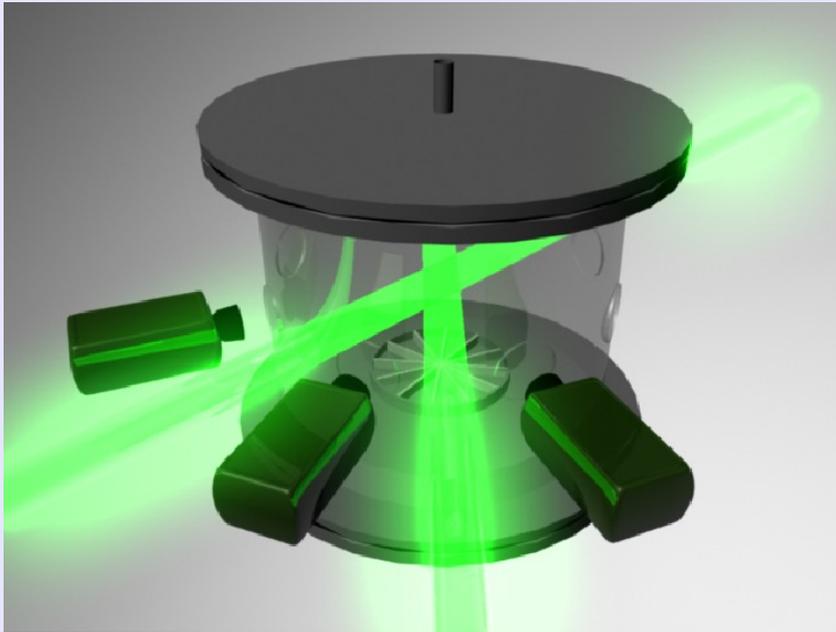
## Turbulenzmodelle

# Kolmogorov



## Energiekaskade

# French Washing Machine

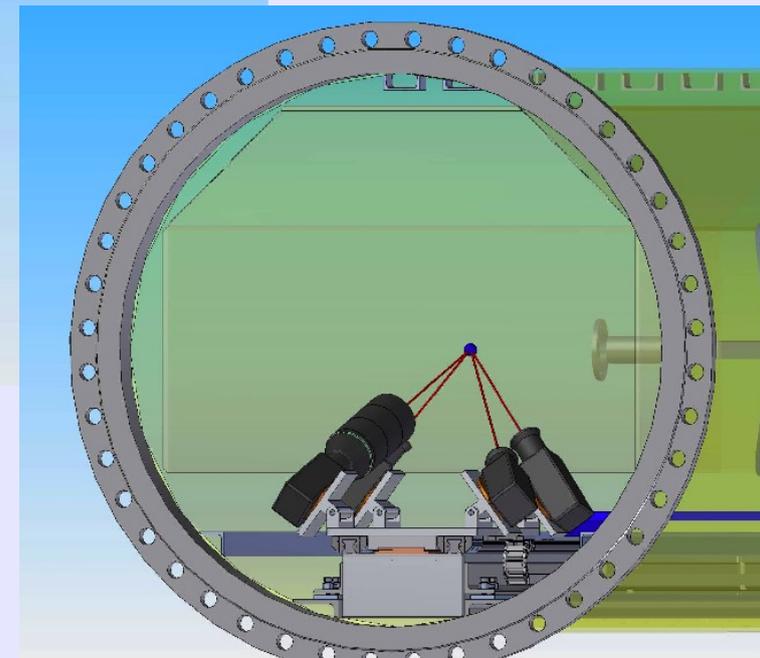
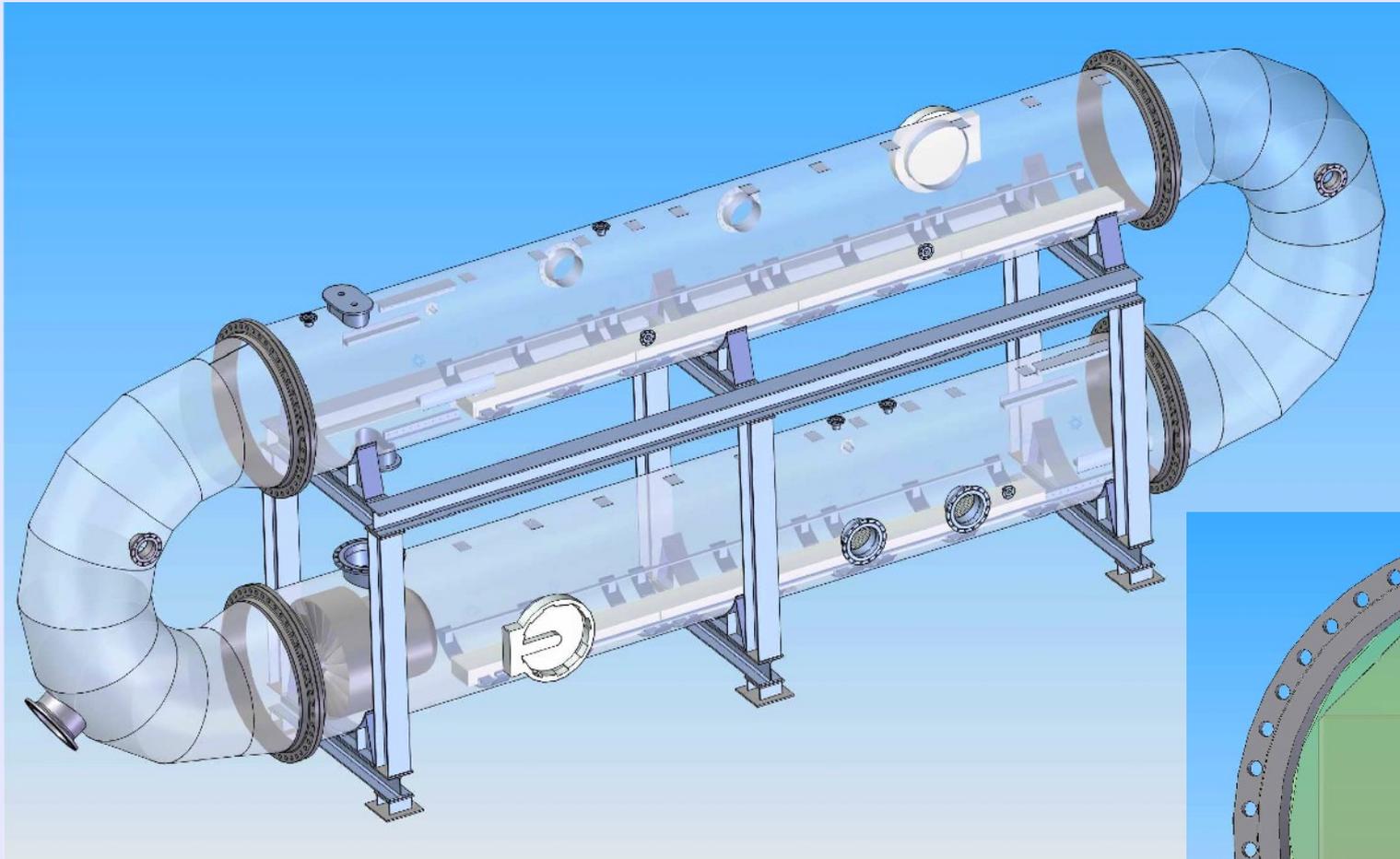


## Turbulenzexperimente

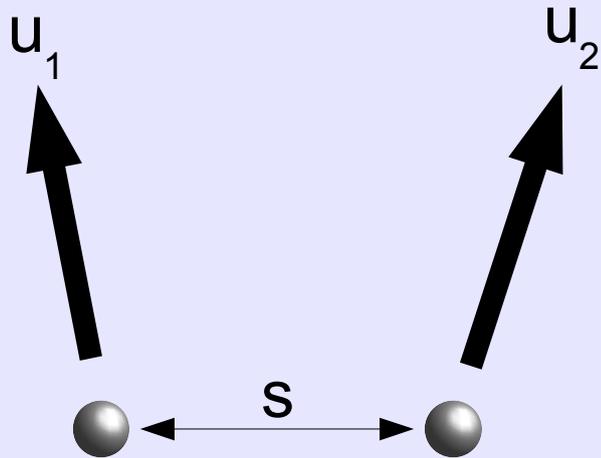
Holger Nobach: Laser-Doppler-Korrelationsmessungen in turbulenten Strömungen



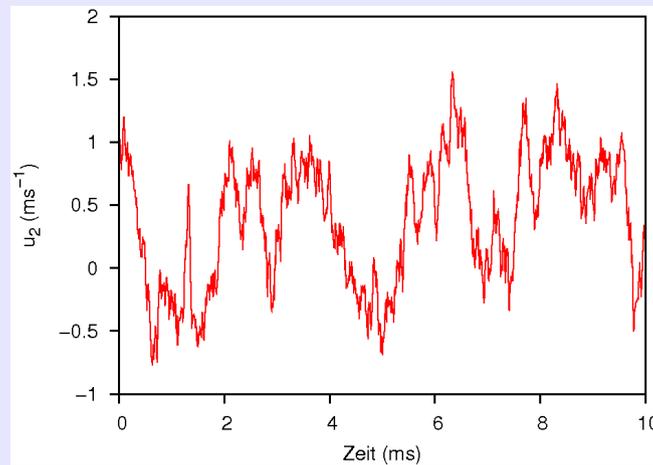
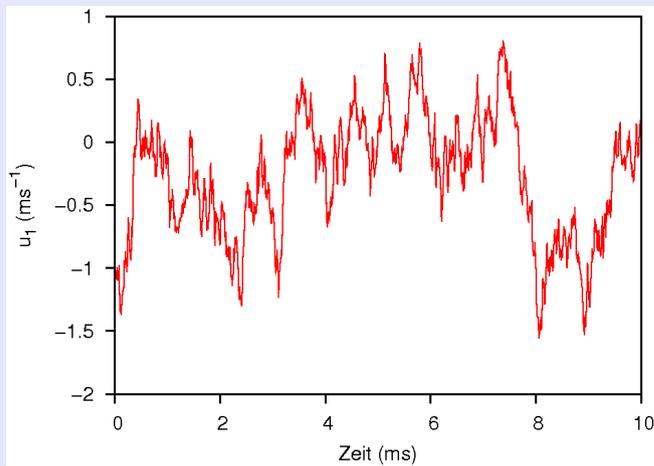
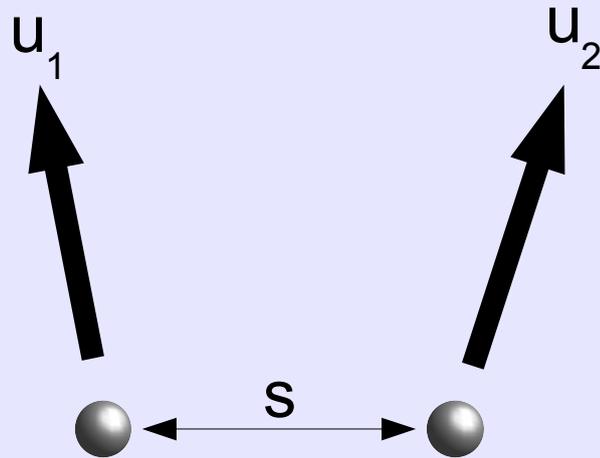
# SF<sub>6</sub>-Druckwindkanal



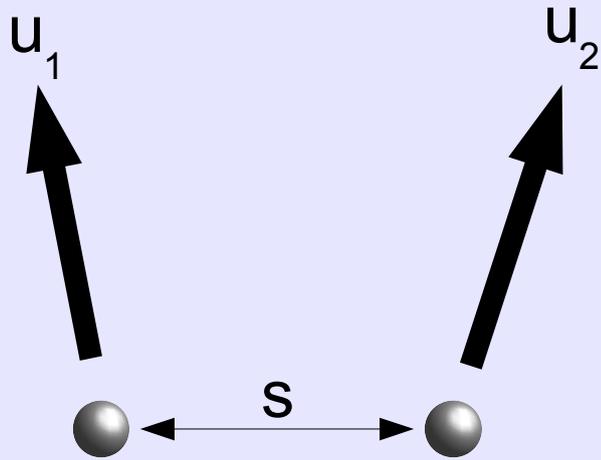
# Korrelationsfunktion



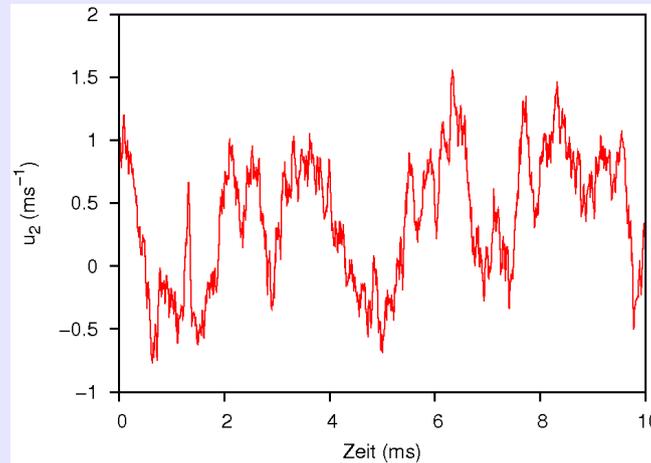
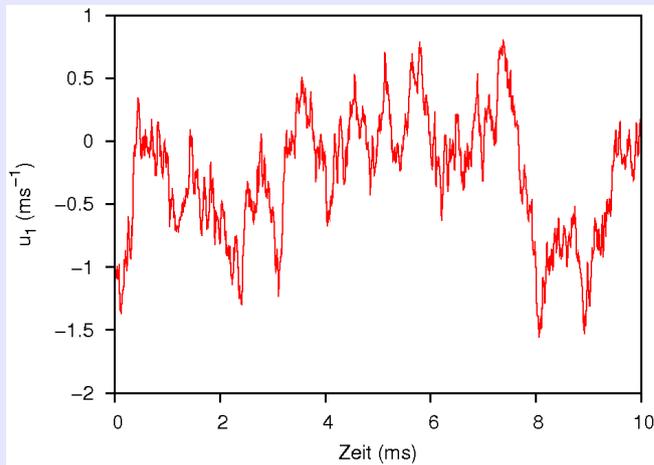
# Korrelationsfunktion



# Korrelationsfunktion

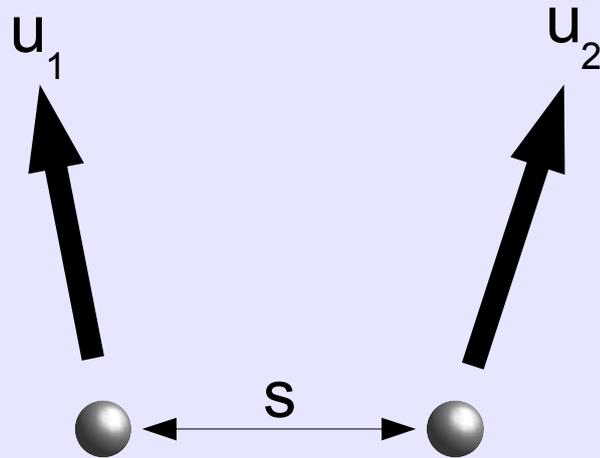


$u_1$	$u_2$	$u_1 \cdot u_2$
+	+	+
-	-	+
+		

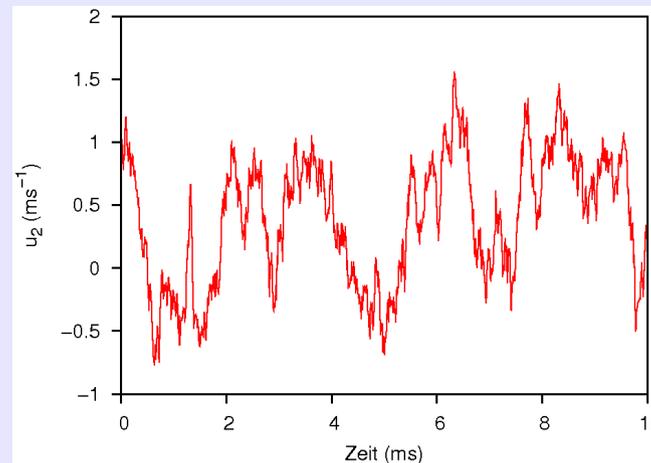
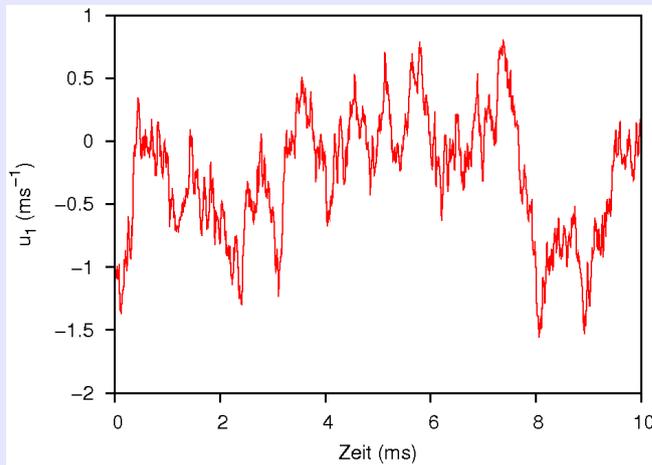


## Korrelationsfunktion

# Korrelationsfunktion



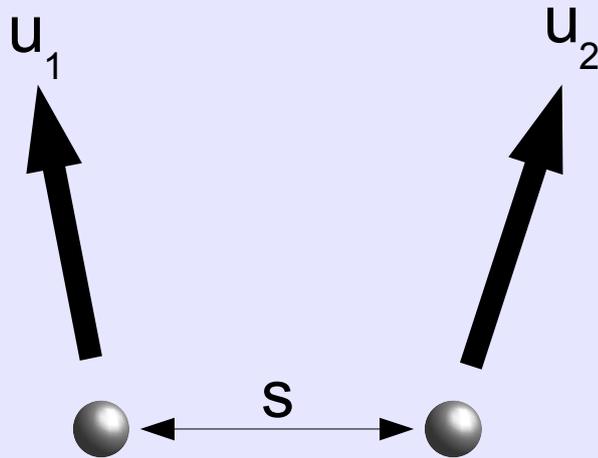
$u_1$	$u_2$	$u_1 \cdot u_2$
+	-	-
-	+	-
<hr/>		-



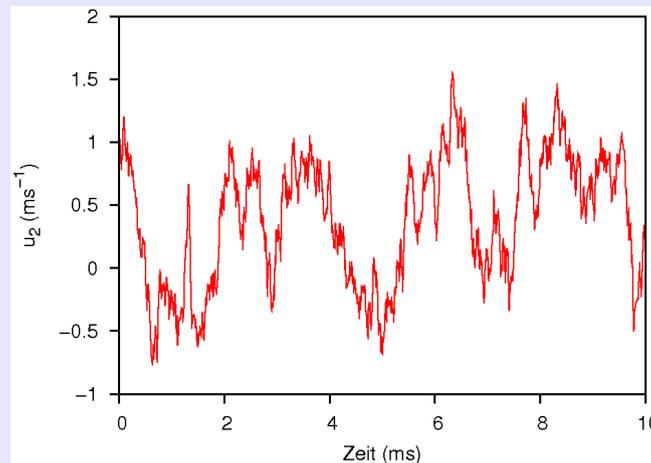
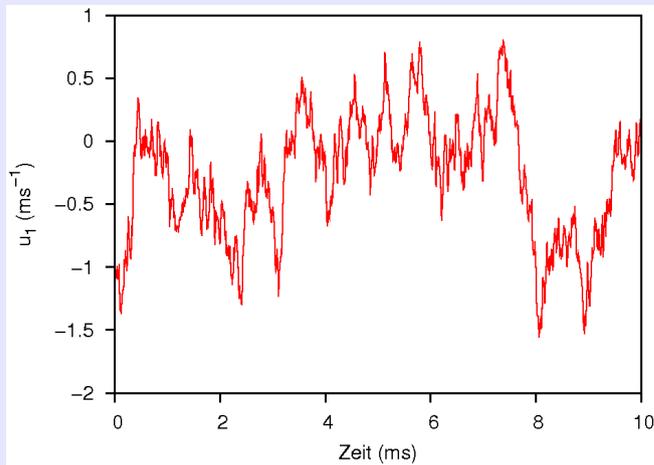
## Korrelationsfunktion



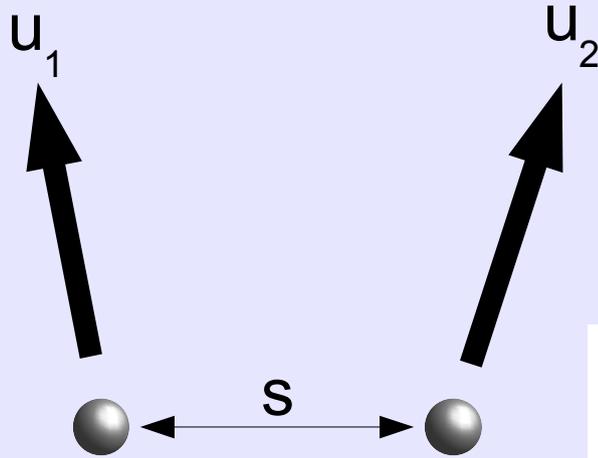
# Korrelationsfunktion



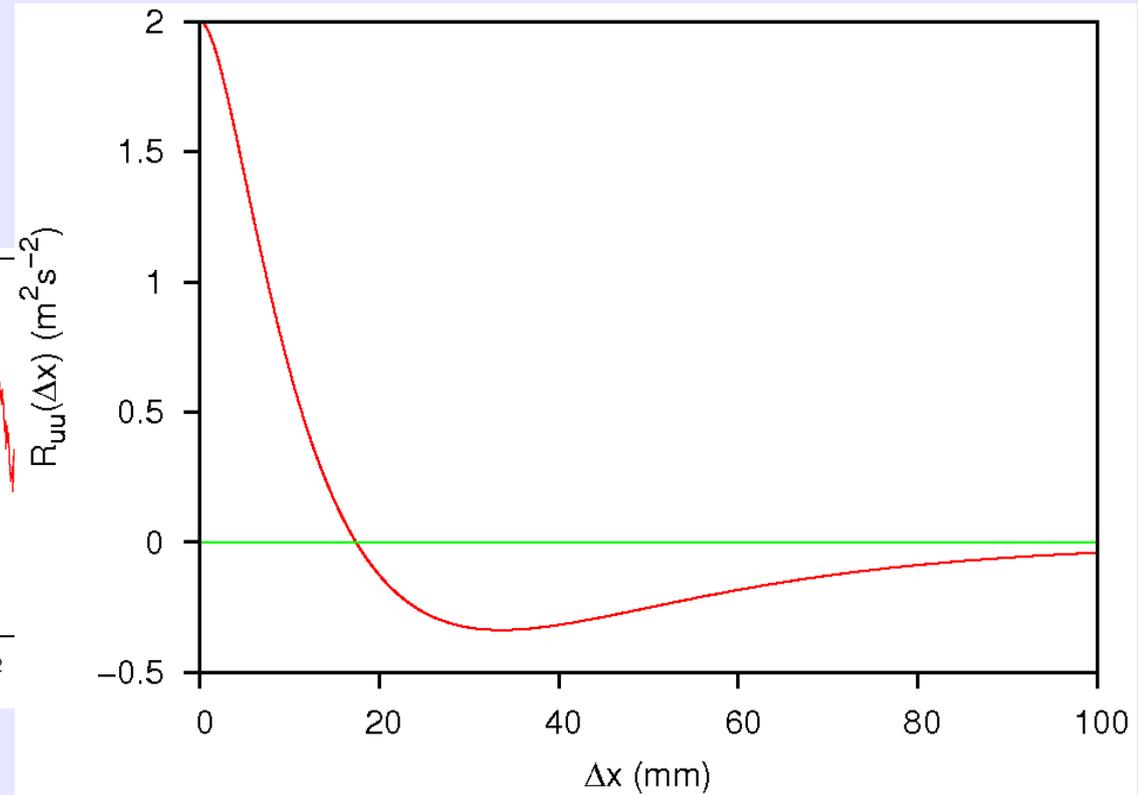
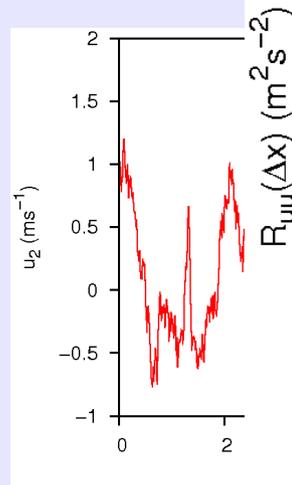
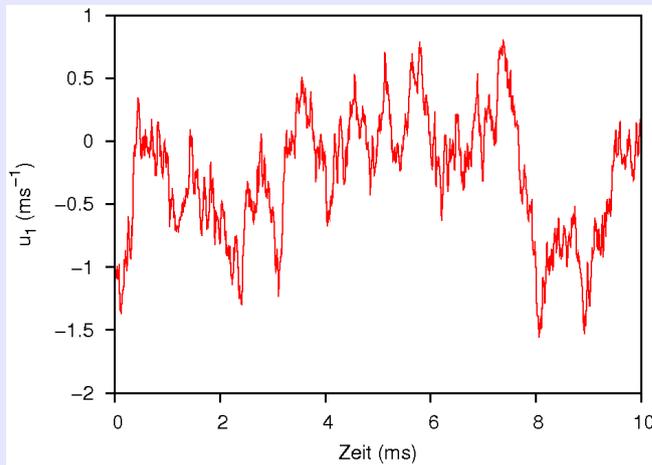
$u_1$	$u_2$	$u_1 \cdot u_2$
+	+	+
+	-	-
-	+	-
-	-	+
0		



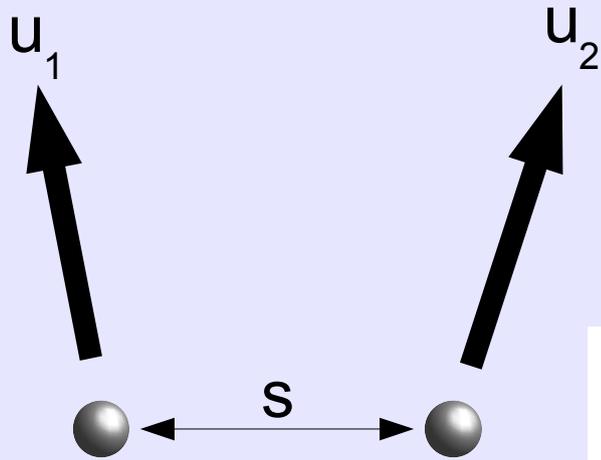
# Korrelationsfunktion



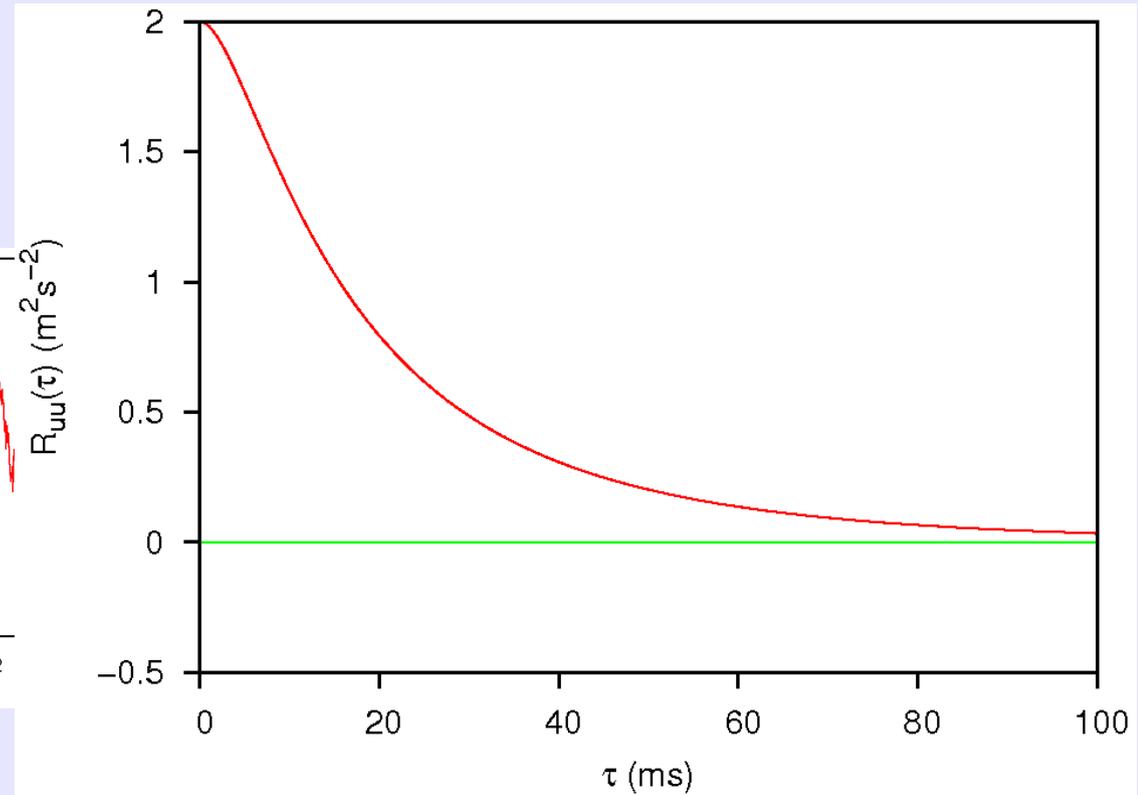
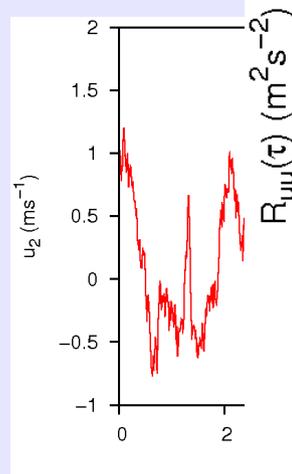
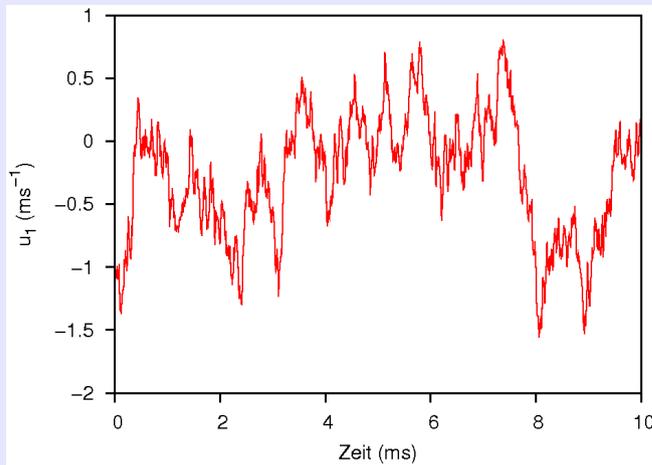
$$R_{u_1 u_2}(\Delta x) = E\{u_1(x) u_2(x + \Delta x)\}$$



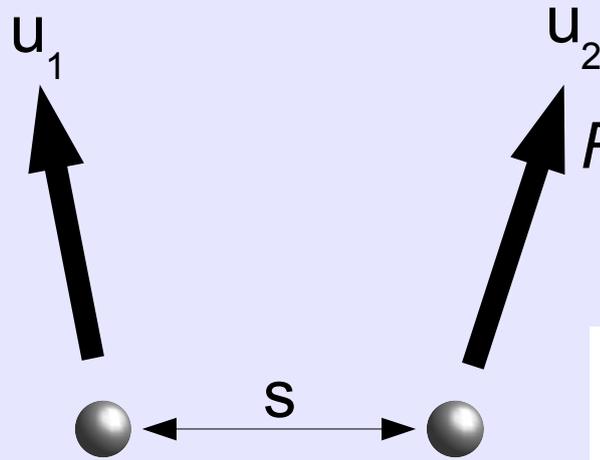
# Korrelationsfunktion



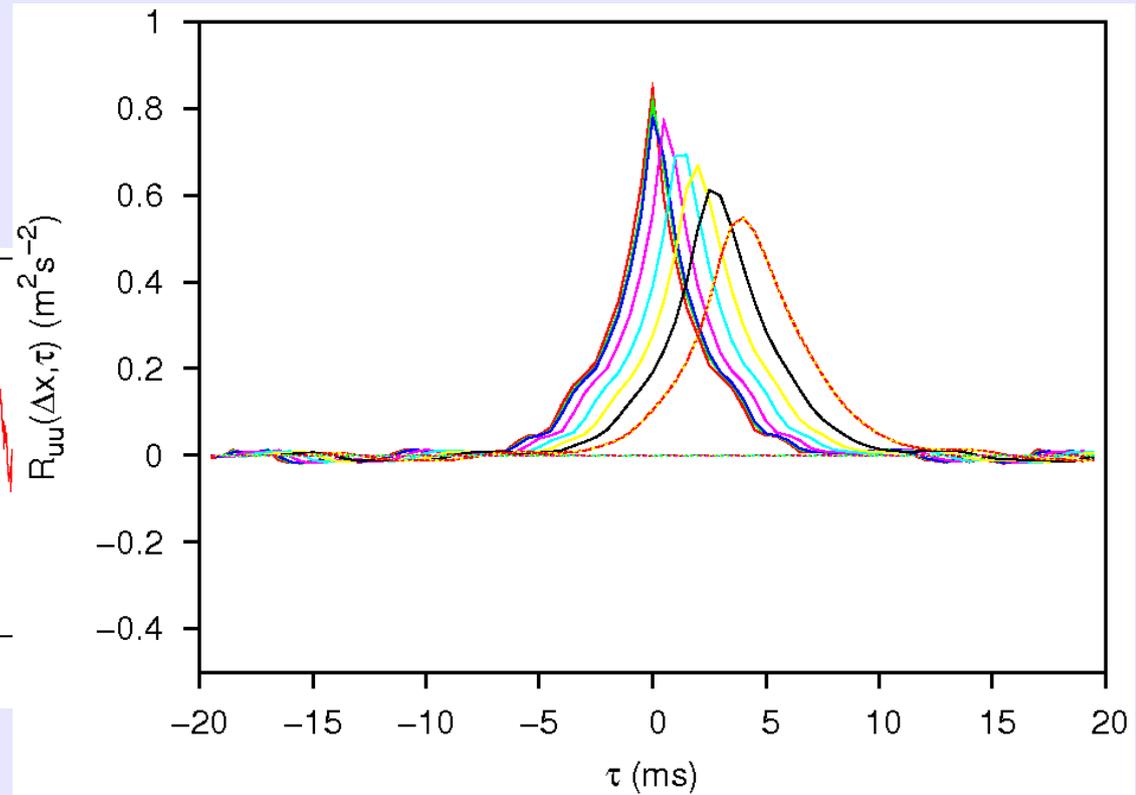
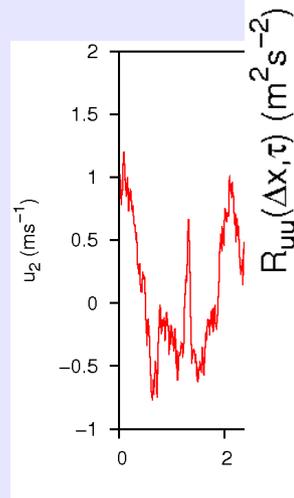
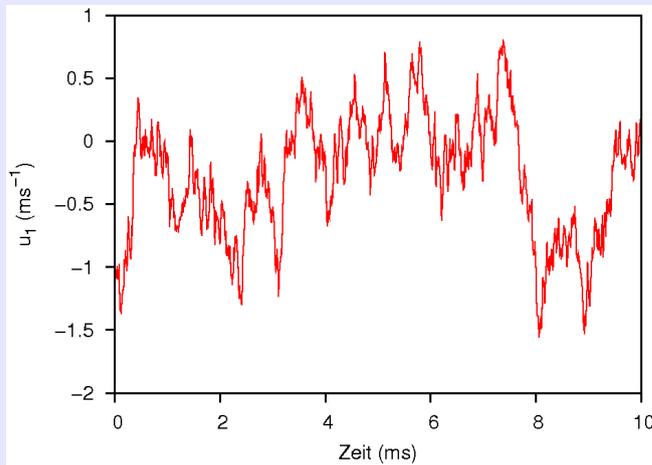
$$R_{u_1 u_2}(\tau) = E\{u_1(t) u_2(t + \tau)\}$$



# Korrelationsfunktion



$$R_{u_1 u_2}(\Delta x, \tau) = E\{u_1(x, t) u_2(x + \Delta x, t + \tau)\}$$

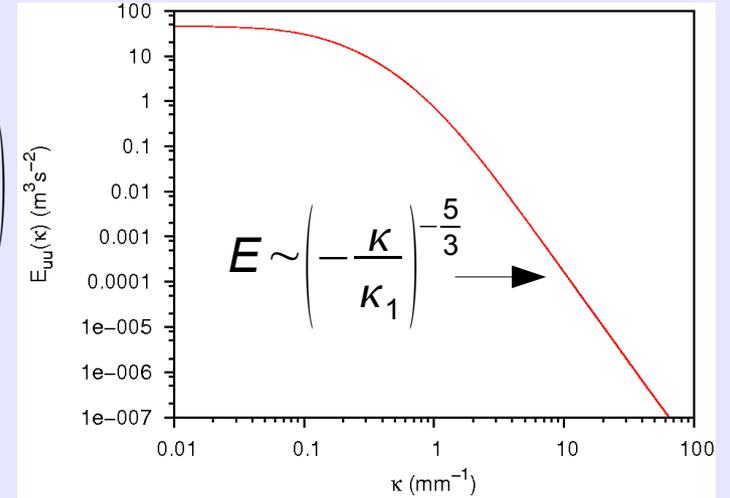
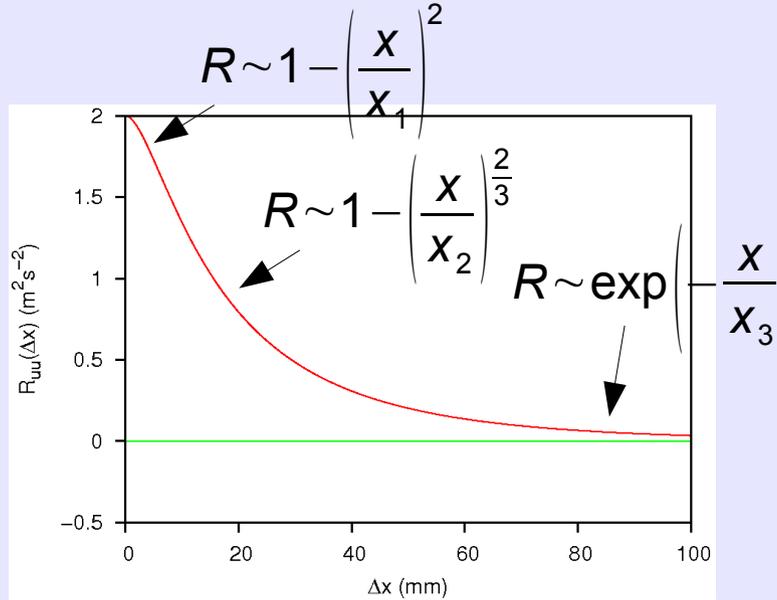


## Korrelationsfunktion

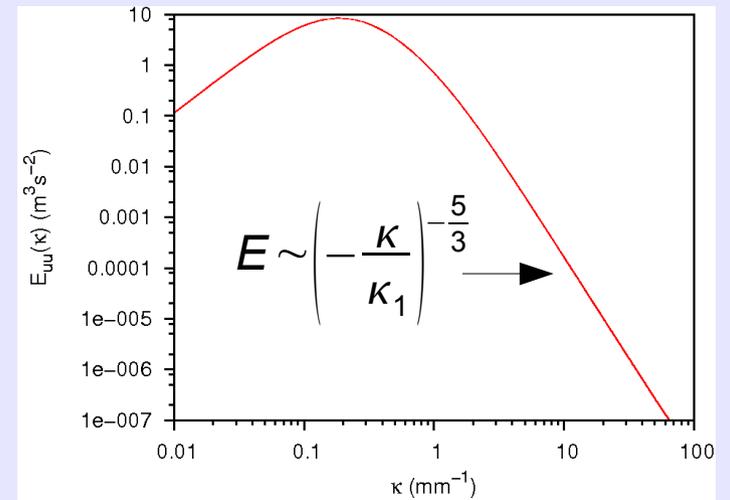
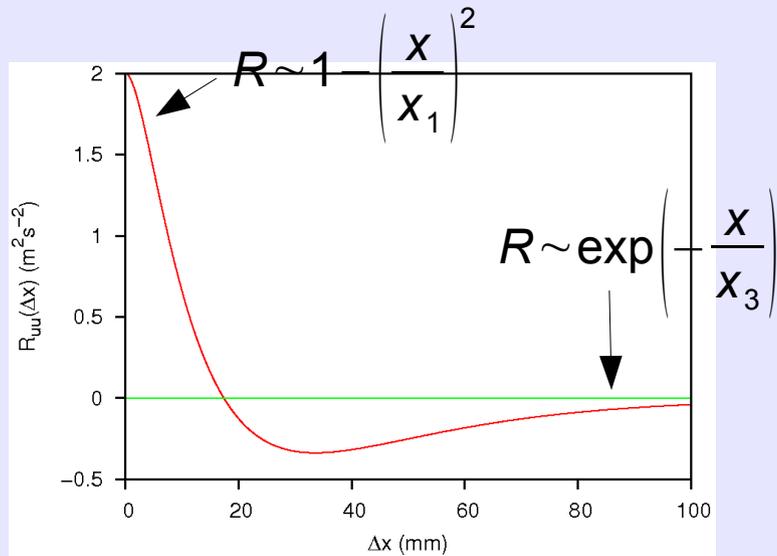
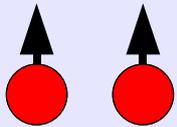


# Turbulenzspektrum

## f-Korrelation



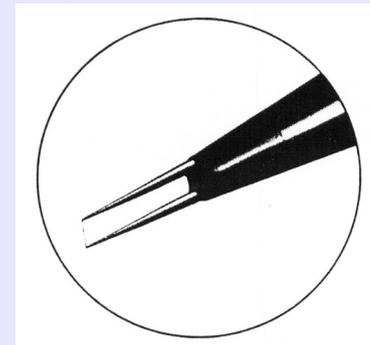
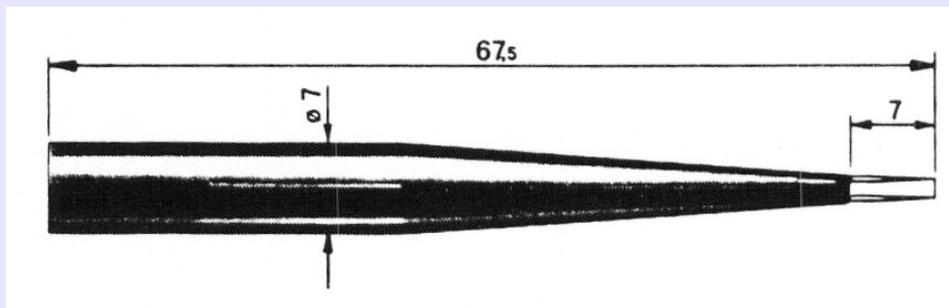
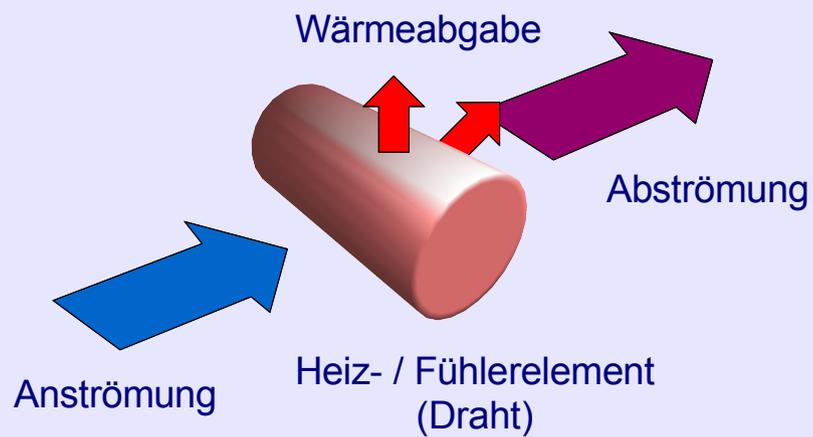
## g-Korrelation



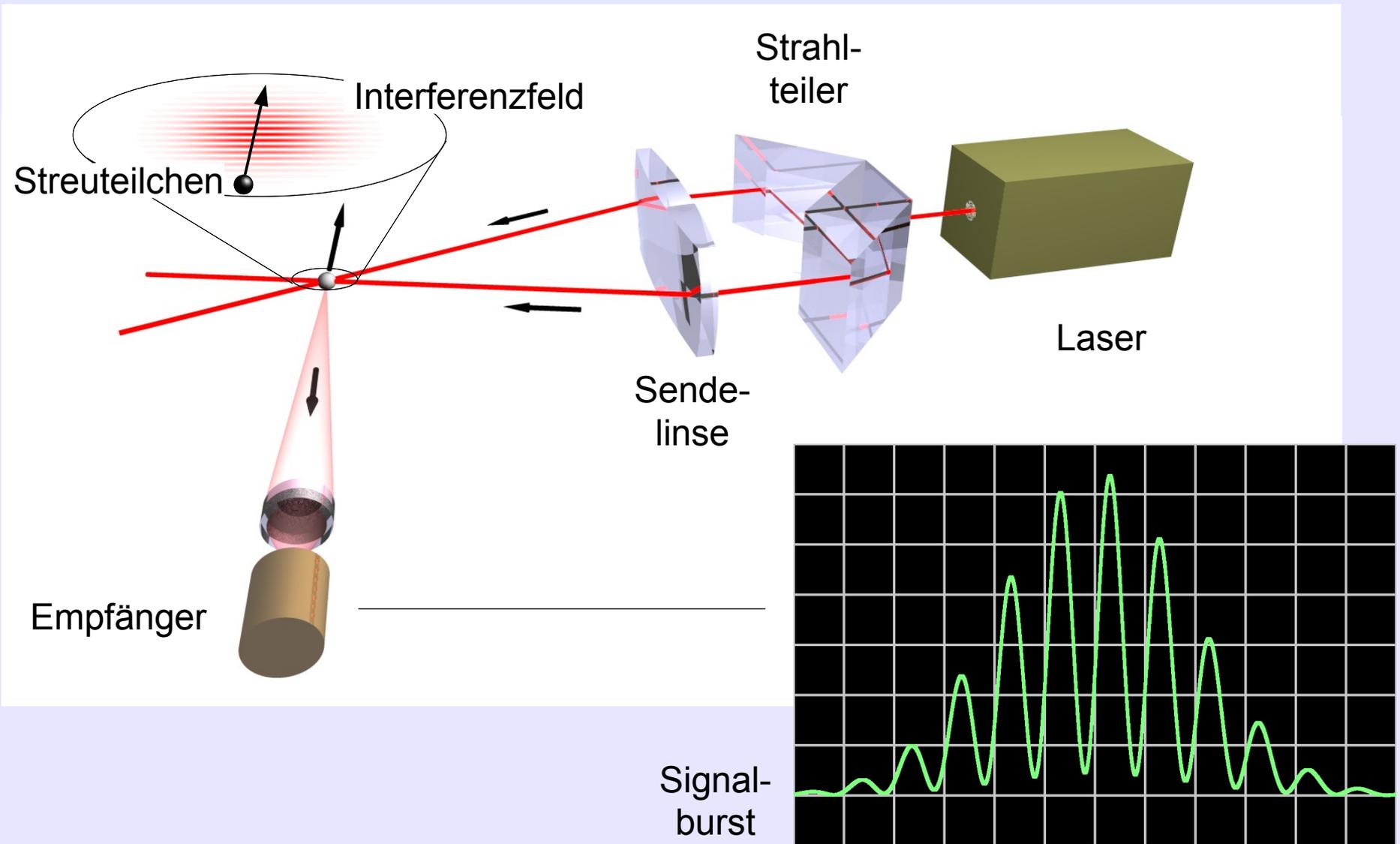
# Turbulenzspektrum



# Hitzdrahtanemometer

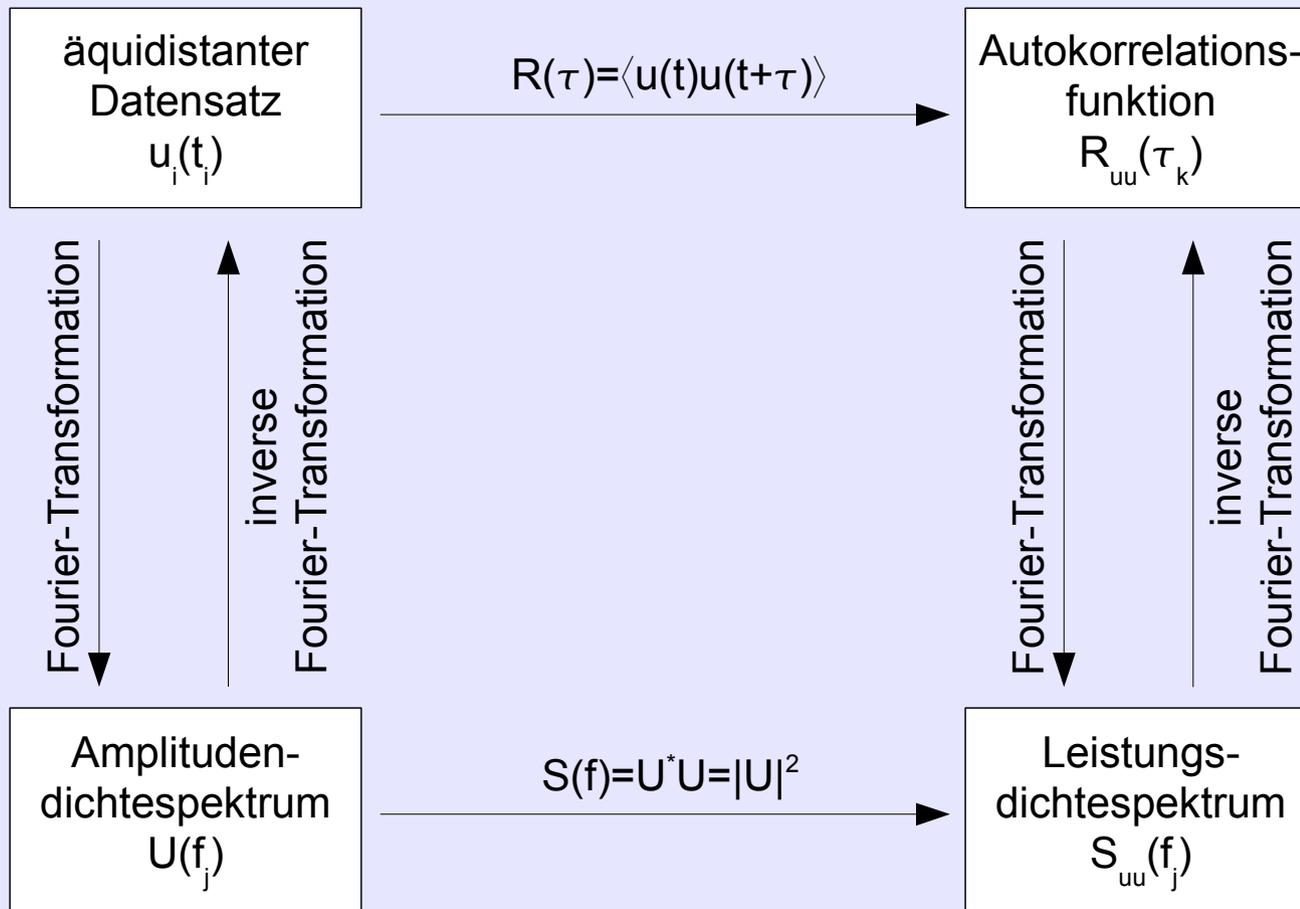
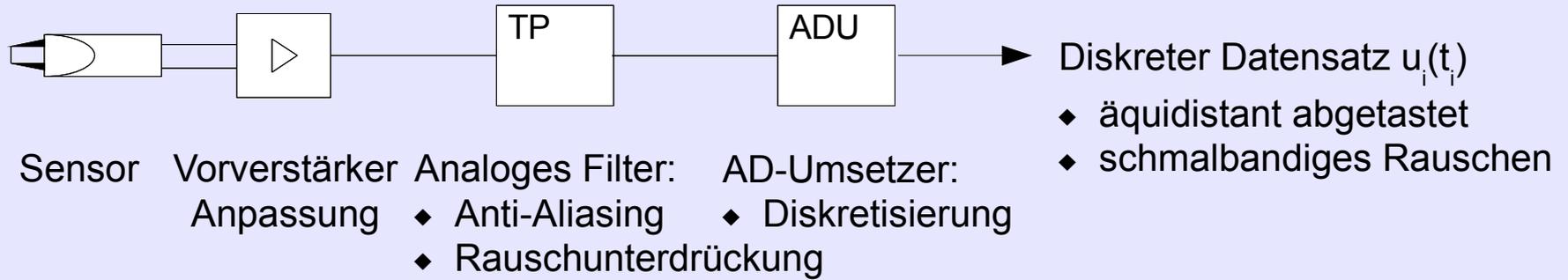


# Laser-Doppler-Anemometer



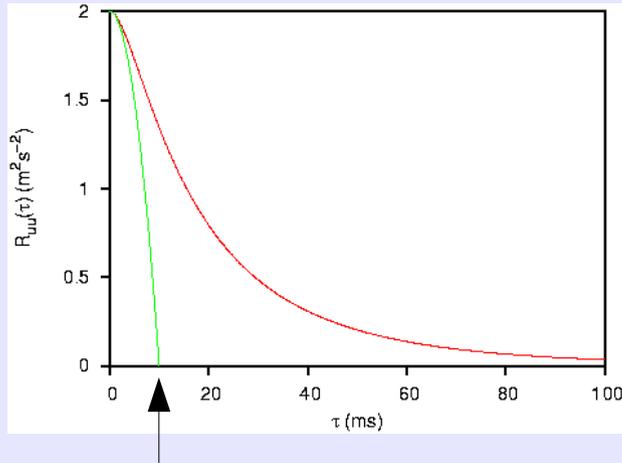
# Datensatzaufnahme und -verarbeitung in der Strömungsmesstechnik

## Hitzdrahtanemometer



# Zeitliche Kennfunktionen

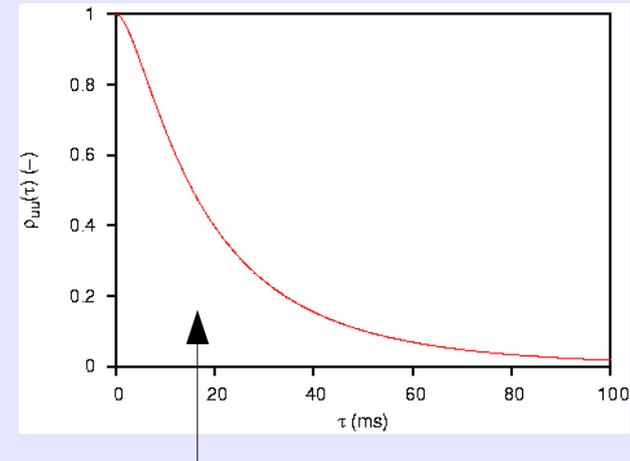
## zeitliche Autokorrelationsfunktion



Taylor-Zeitmaß  $T_\lambda$

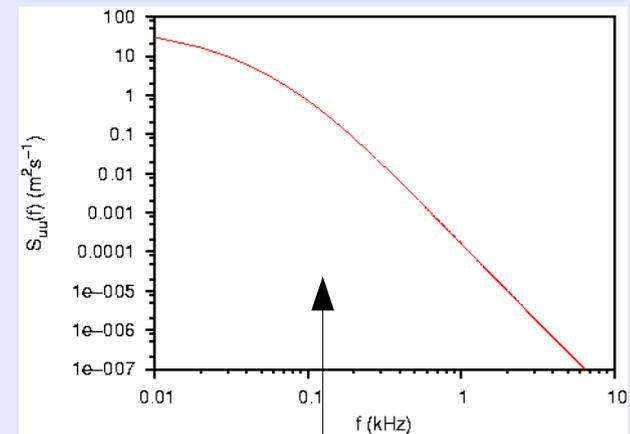
Normierung

## Korrelationskoeffizient



integrales Zeitmaß  $T_L$

## spektrale Leistungsdichte



Varianz

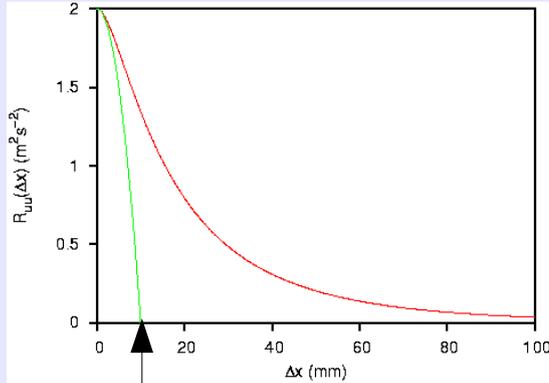
Fourier-Transformation

# Zeitliche Kennfunktionen



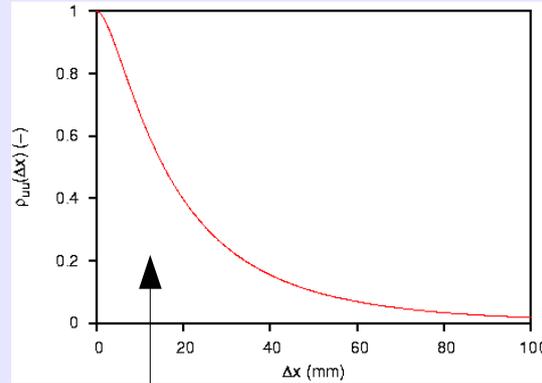
# Räumliche Kennfunktionen

räumliche  
Korrelationsfunktion



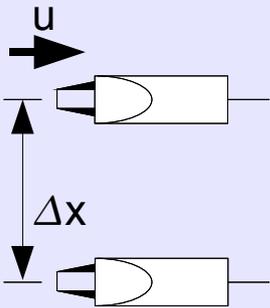
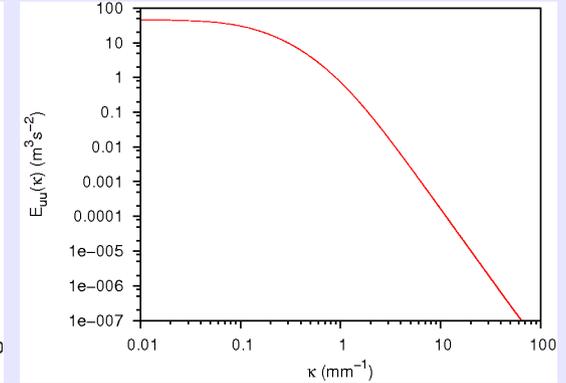
Taylor-Mikromaß  $\lambda_f$

räumlicher  
Korrelationskoeffizient

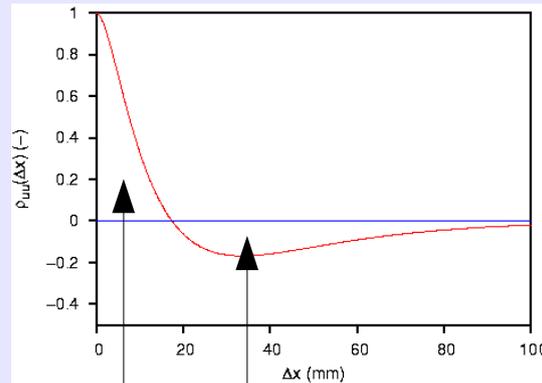


integrales Längenmaß  $L_f$

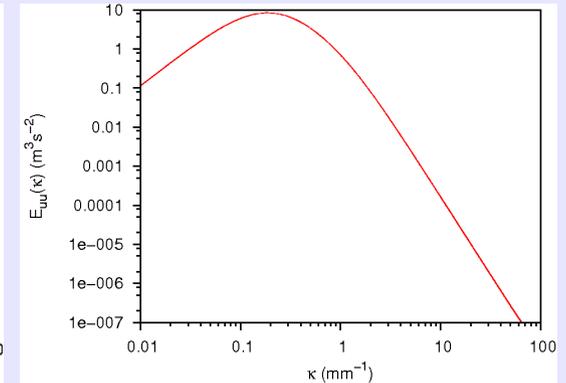
Wellenzahlspektrum



Taylor-Mikromaß  $\lambda_g$



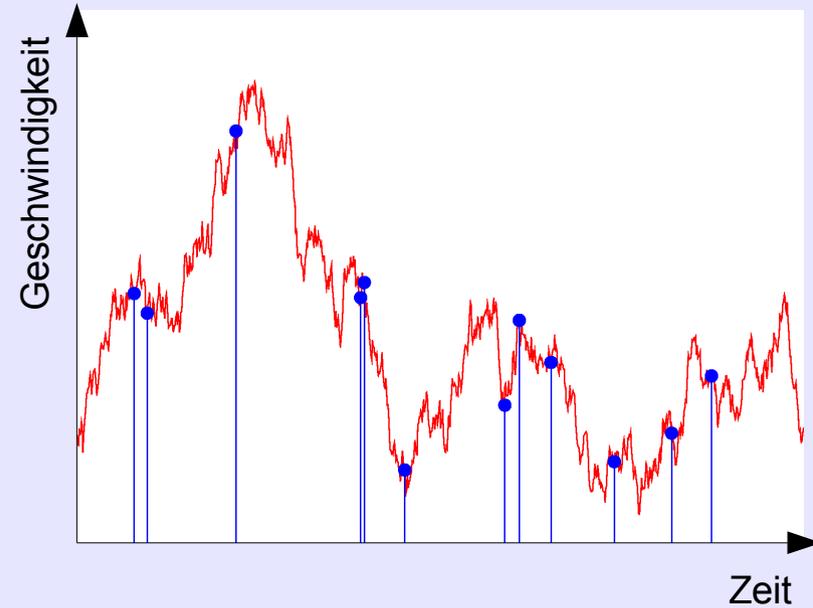
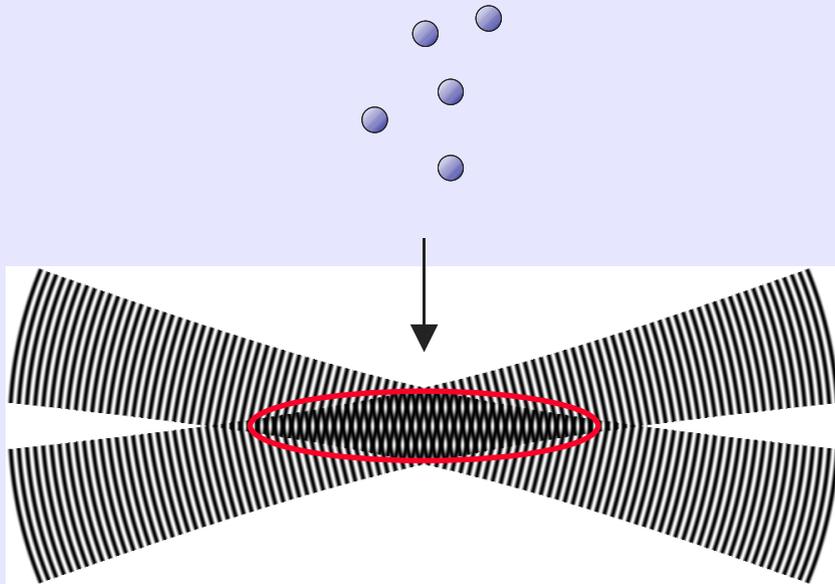
integrales Längenmaß  $L_g$



Abschätzung der Dissipationsrate  $\epsilon$

## Räumliche Kennfunktionen

# LDA-Datensatz



Einzelteilchenmessung

Unsicherheit der Frequenzschätzung

Korrelation zw. Teilchenrate und Geschwindigkeit

Interferenz des Streulichtes verschiedener Teilchen

⇒ zufällig abgetastete Zeitreihe

⇒ breitbandiges Rauschen

⇒ Korrelation zw. Datenrate und Geschwindigkeit

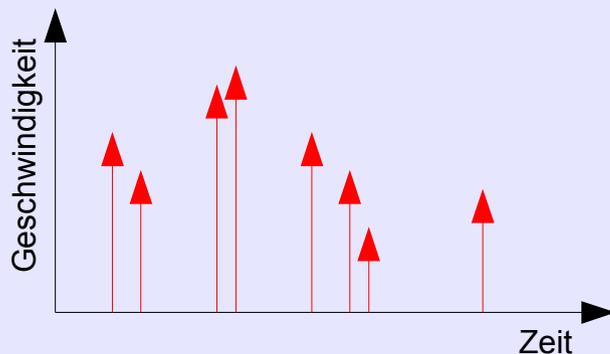
⇒ Prozessortzeit

# Strategien für die Auswertung von unregelmäßig abgetasteten Datensätzen

unregelmäßig abgetasteter Datensatz

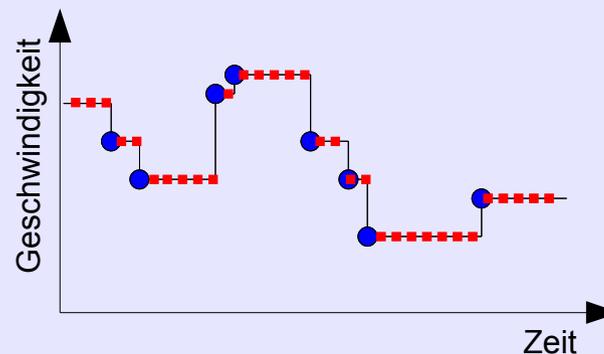
Direkte Verarbeitung

- ◆ Mathematische Beschreibung des Signals (z.B. Folge von Dirac-Impulsen)
- ◆ Berücksichtigung der unregelmäßigen Abtastung
- ◆ Entwicklung geeigneter Schätzer



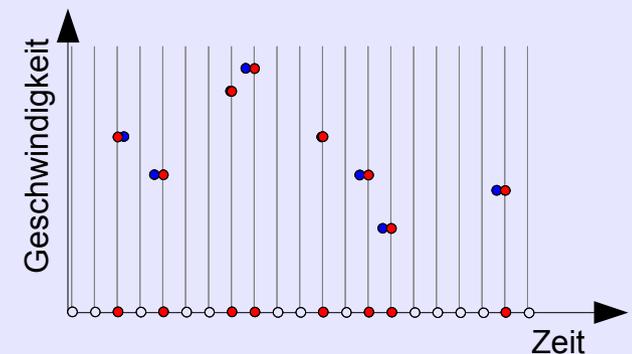
Signalrekonstruktion und regelmäßige Wiederabtastung

- ◆ Wahl einer geeigneten Rekonstruktions- bzw. Interpolationsvorschrift
- ◆ Klassische Datenverarbeitung



Transformation in einen dünn besetzten Datensatz

- ◆ Quantisierung der Abtastzeitpunkte oder -intervalle
- ◆ Berücksichtigung von Signallücken
- ◆ Schätzer aus der Prozessidentifikation

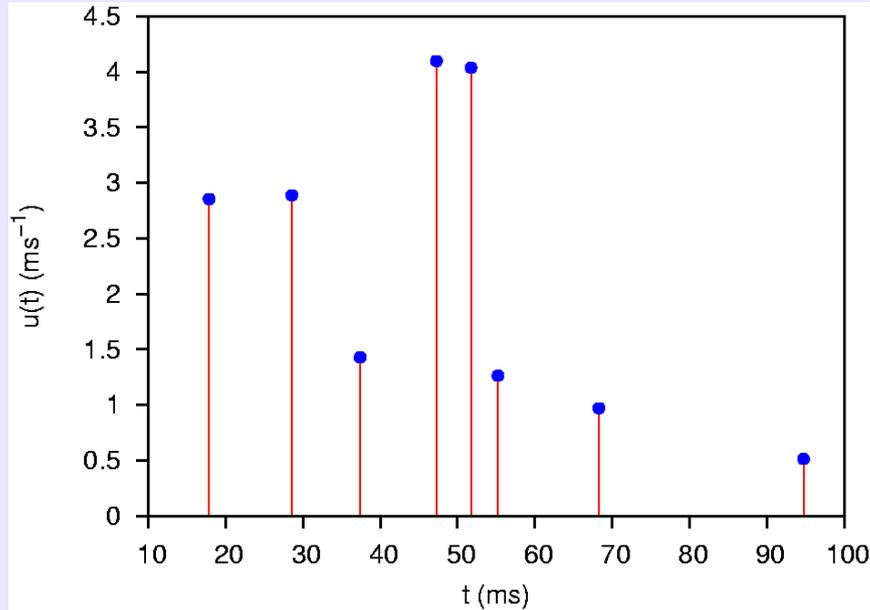


Auswertestrategien

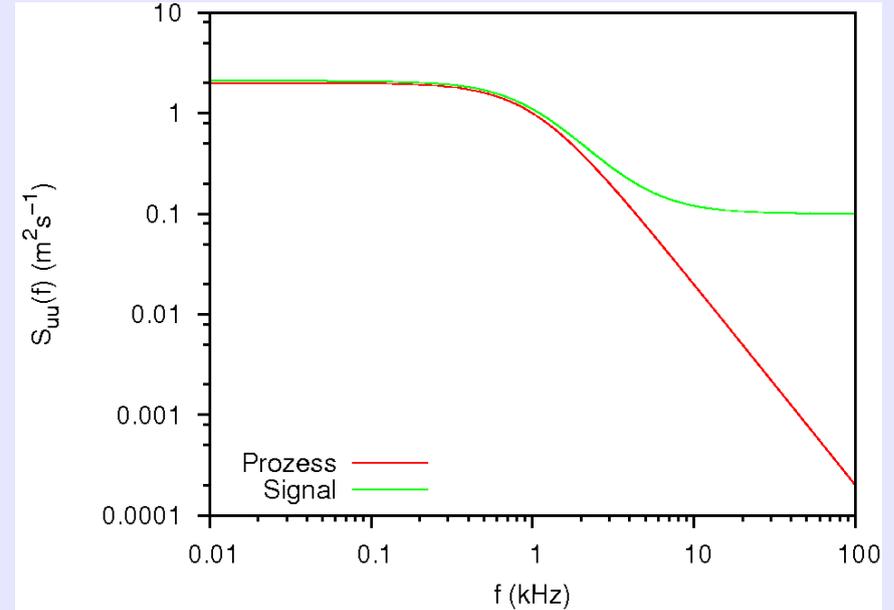


MPIDS

◆ Prinzip der direkten Spektralanalyse



◆ Systematischer Fehler aufgrund der unregelmäßigen Abtastung



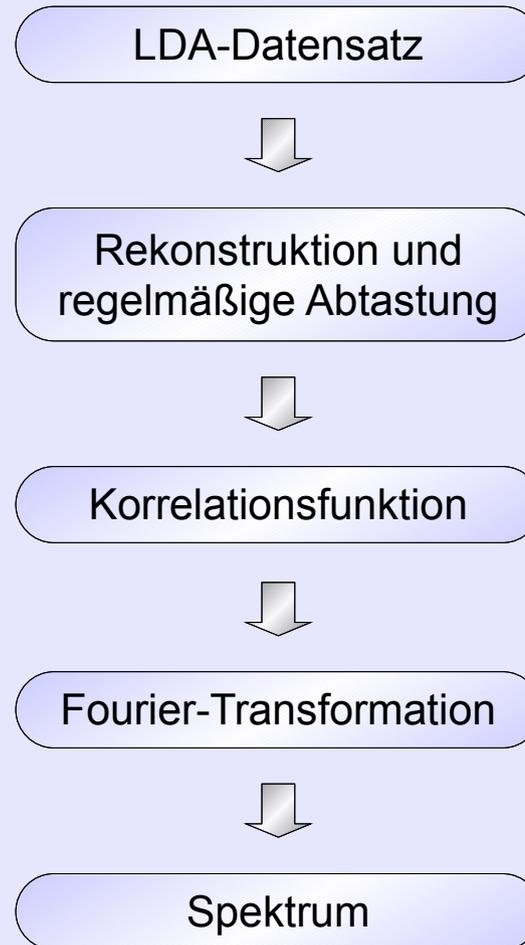
$$S_S(f) \stackrel{\text{def}}{=} \frac{1}{T} \left| \int_0^T u(t) e^{-2\pi j f t} dt \right|^2 = \frac{T}{N^2} \left| \sum_{i=1}^N u_i e^{-2\pi j f t_i} \right|^2$$

Fehlerabschätzung:  $E\{S_S\} = S_P + \frac{T}{N} \sigma_u^2$

Korrektur:  $\hat{S}_P(f) = \frac{T}{N^2} \left( \left| \sum_{i=1}^N u_i e^{-2\pi j f t_i} \right|^2 - \sum_{i=1}^N u_i^2 \right)$

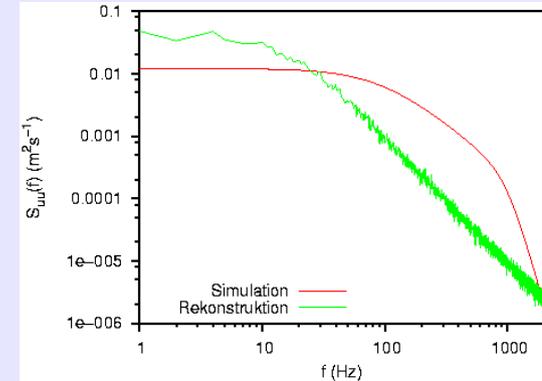
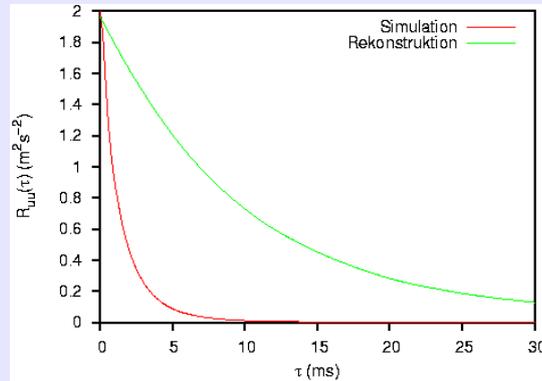
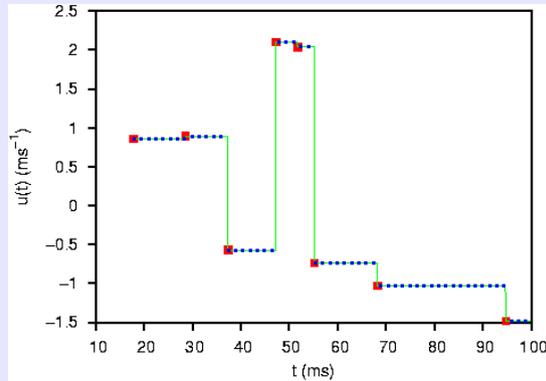
[GAS77]

## ◆ Ablauf

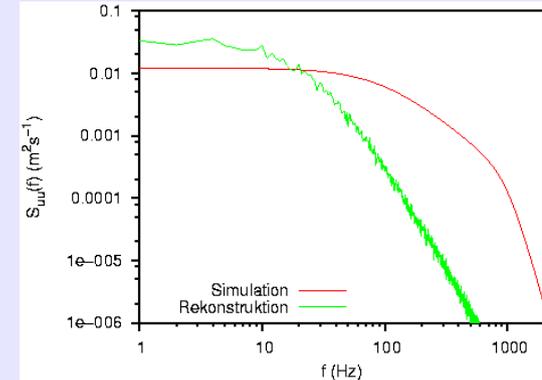
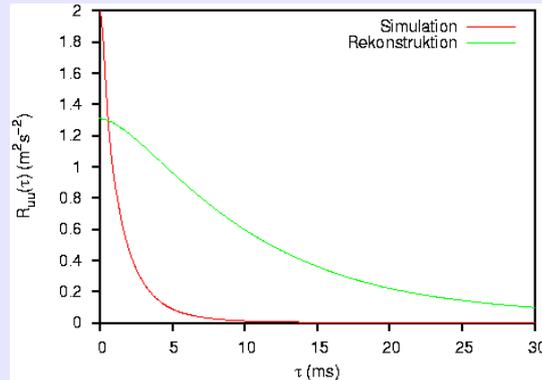
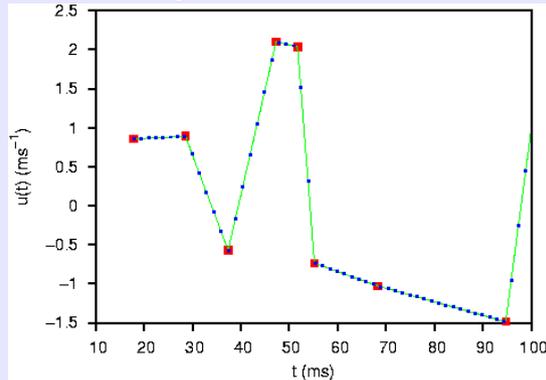


# Auswirkung der Rekonstruktion

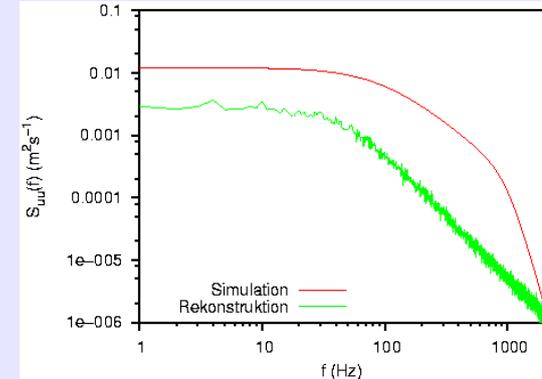
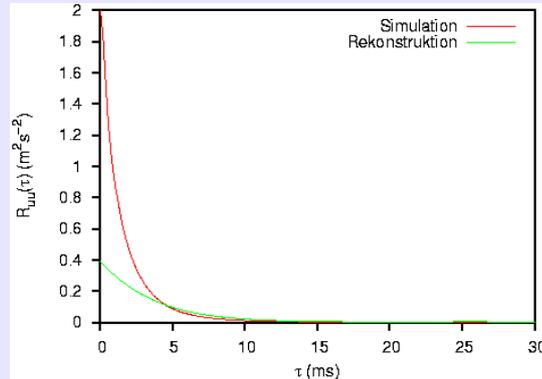
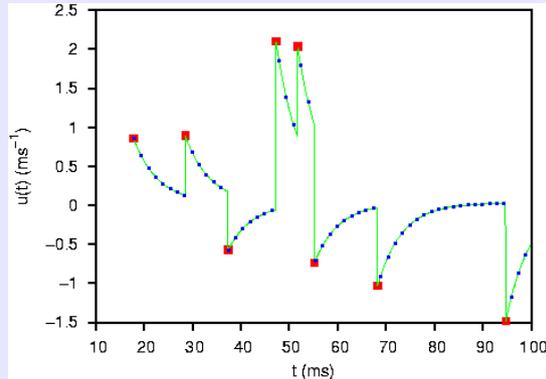
## ◆ Sample-and-Hold-Rekonstruktion



## ◆ lineare Interpolation



## ◆ exponentielle Rekonstruktion [HOS94]

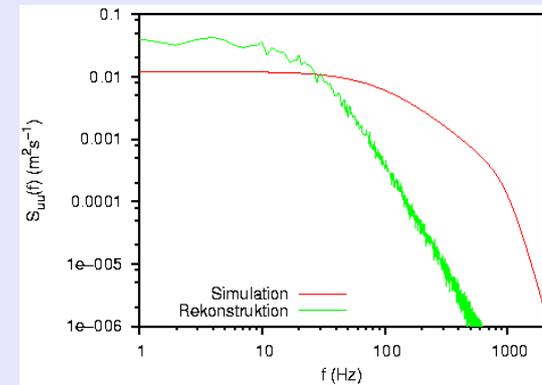
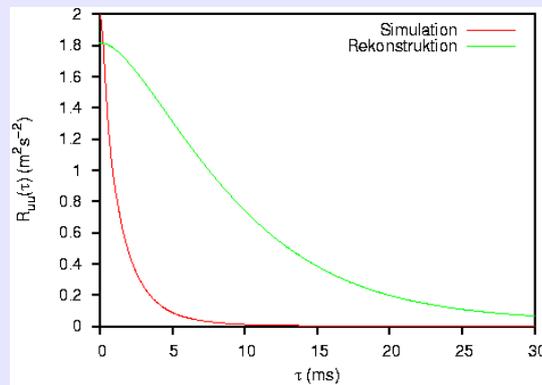
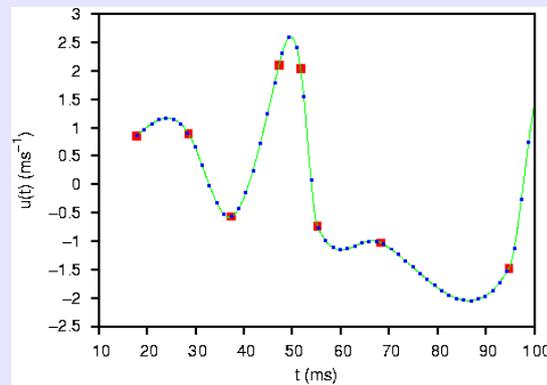


# Tiefpasswirkung der Rekonstruktion



# Auswirkung der Rekonstruktion

## ◆ Spline-Interpolation

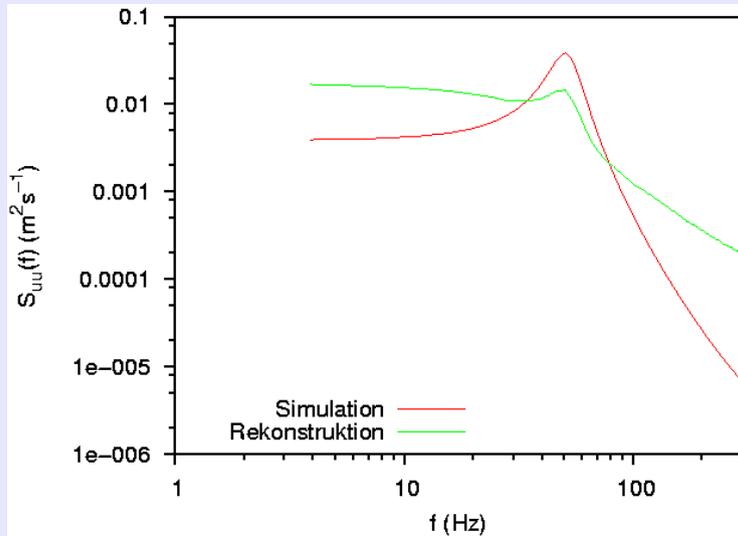
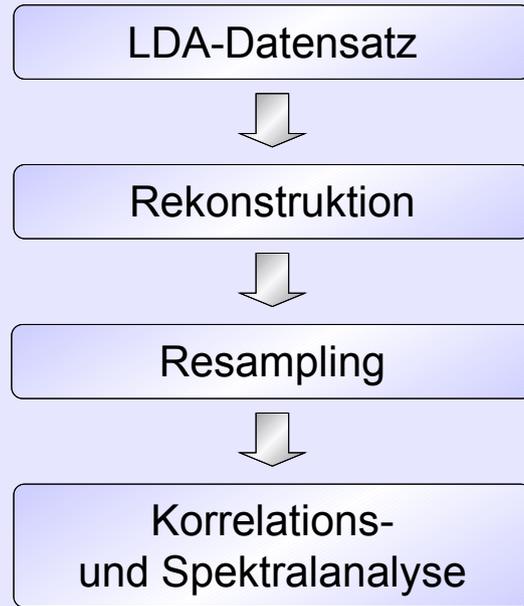


- ◆ Kalman-Rekonstruktion [BEN95,MAA94]
- ◆ Shannon-Rekonstruktion [CLA85,VEY88]
- ◆ Anpassung einer bandbegrenzten Funktion (POCS) [LEE92,KUO92,SAU87,YEH90]
- ◆ fraktale Rekonstruktion [STR88,STR91,CHA92]

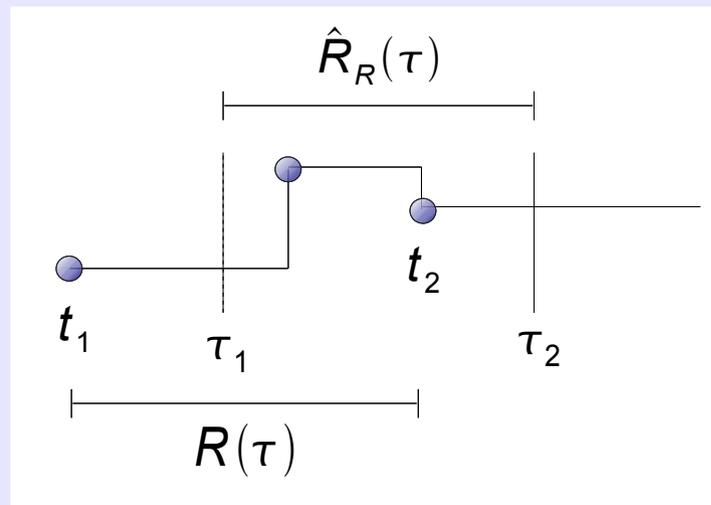
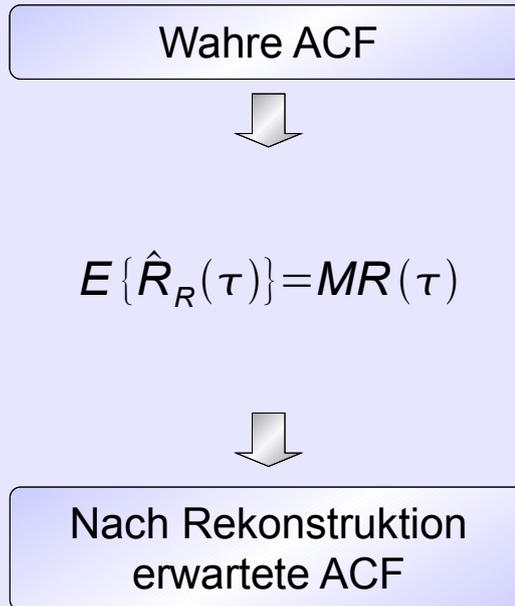
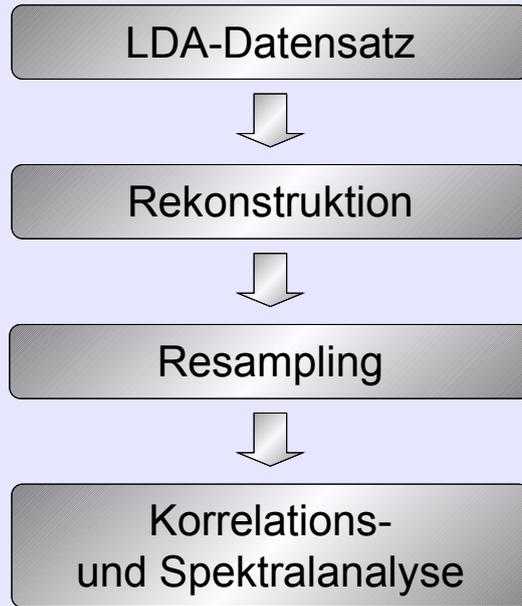
Allen Rekonstruktionen (unabhängig von der Rekonstruktionsvorschrift) ist gemeinsam:

- ◆ Bei hoher Datenrate sind alle Verfahren geeignet, aus dem unregelmäßig abgetasteten LDA-Datensatz einen regelmäßig abgetasteten Datensatz zu erzeugen, der die spektralen Eigenschaften des Strömungsprozesses widerspiegelt.
- ◆ Bei geringer Datenrate verändern sich die spektralen Eigenschaften. Der spektrale Charakter des Rekonstruktionsergebnisses wird direkt und unabhängig vom zugrundeliegenden Strömungsprozess von der verwendeten Rekonstruktionsvorschrift und der Datenrate bestimmt.

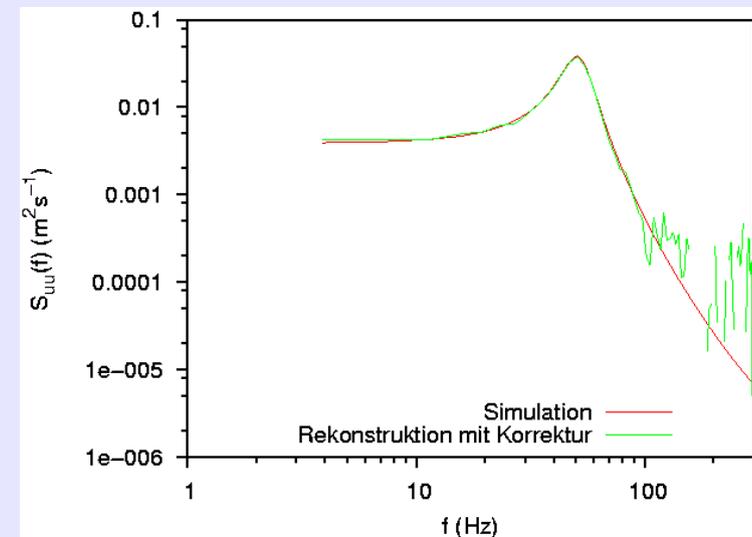
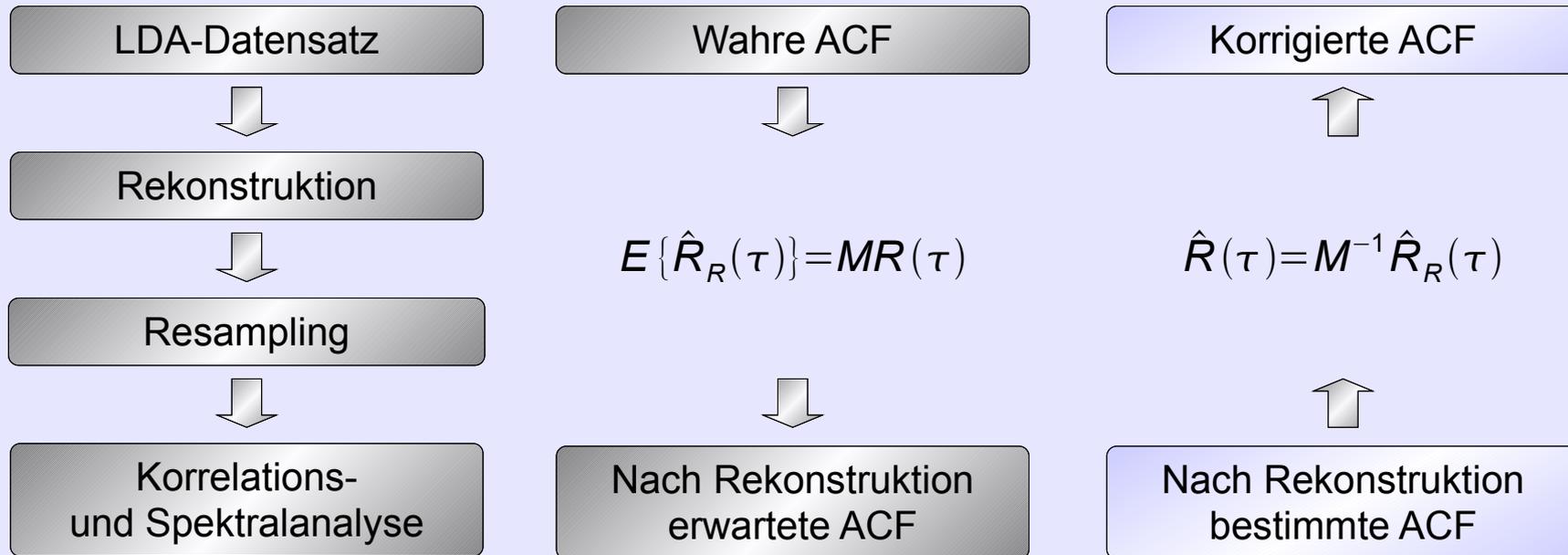
## 1. Analyse des rekonstruierten Datensatzes



## 2. Abschätzung des Filters



### 3. Korrektur



- ◆ Sample-and-Hold-Rekonstruktion

- ◆ Rekonstruktionsvorschrift
- ◆ Interpolationsfilter

$$u_R(t) = u_i \quad t_i \leq t < t_{i+1}$$

$$E\{\hat{R}_R(\tau_k)\} = e^{-\dot{n}\tau_k} \left\{ R(0) + \frac{(e^{\dot{n}\Delta\tau} - 1)^2}{1 - e^{2\dot{n}\Delta\tau}} \sum_{\xi=1}^{\infty} e^{-\dot{n}\tau_\xi} (1 - e^{2\dot{n}\min(k,\xi)\Delta\tau}) R(\tau_\xi) \right\}$$

- ◆ Korrektur

$$\hat{R}(\tau_k) = \begin{cases} \hat{R}_R(0) & \text{für } k=0 \\ (2c+1)\hat{R}_R(\tau_k) - c[\hat{R}_R(\tau_{k-1}) + \hat{R}_R(\tau_{k+1})] & \text{sonst} \end{cases} \quad c = \frac{e^{-\dot{n}\Delta\tau}}{(1 - e^{-\dot{n}\Delta\tau})^2}$$

- ◆ Proportional-Ein-Punkt-Rekonstruktion (exp., Korrelationskoeffizient, S&H)

- ◆ Rekonstruktionsvorschrift
- ◆ Interpolationsfilter

$$u_R(t) = u_i f_R(t - t_i) \quad t_i \leq t < t_{i+1}$$

$$E\{\hat{R}_R(\tau_k)\} = R(0) \sum_{i=-\infty}^0 f_R(-\tau_i) f_R(\tau_k - \tau_i) (1 - e^{-\dot{n}\Delta\tau}) e^{-\dot{n}(\tau_k - \tau_i)} + \sum_{\xi=1}^{\infty} R(\tau_\xi) \sum_{i=1}^{\min(k,\xi)} f_R(\tau_\xi - \tau_i) f_R(\tau_k - \tau_\xi) (1 - e^{-\dot{n}\Delta\tau})^2 e^{-\dot{n}(\tau_k - 2\tau_i + \tau_\xi)}$$

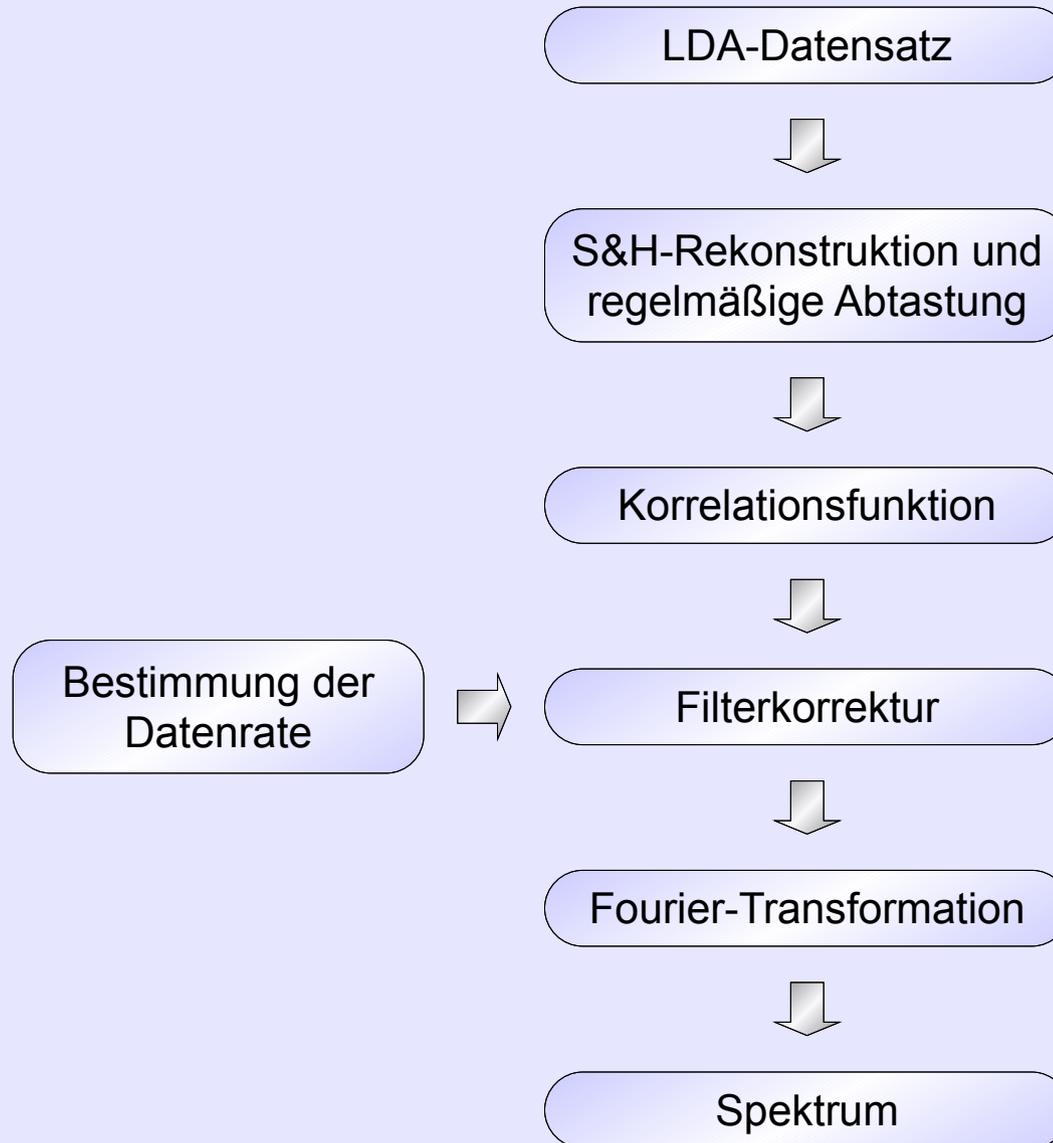
- ◆ Korrektur erfolgt numerisch durch Lösung des linearen Gleichungssystems

- ◆ andere Interpolationen

- ◆ prinzipiell auch für andere Interpolationen geeignet
- ◆ numerischer Aufwand steigt mit der Anzahl der verwendeten Stützstellen stark an
- ◆ geringer Gewinn gegenüber Sample-and-Hold-Interpolation



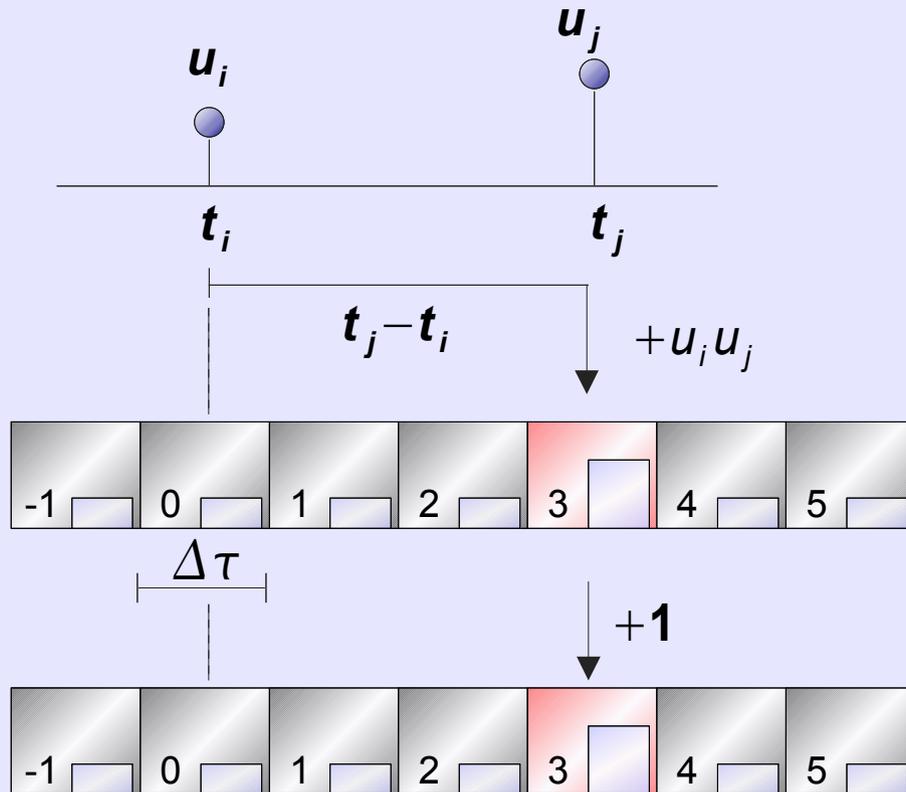
◆ Ablauf mit Filterkorrektur



◆ Grundprinzip [GAS75, MAY74, 78, SCO74]

$$i=1\dots N$$

$$j=1\dots N \quad j \neq i$$



$$z_k = \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N u_i u_j b_k(t_j - t_i)$$

$$n_k = \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N b_k(t_j - t_i)$$

$$b_k(\Delta t) = \begin{cases} 1 & \text{für } (k-1/2)\Delta\tau \leq \Delta t < (k+1/2)\Delta\tau \\ 0 & \text{sonst} \end{cases}$$

AKF  
 $\hat{R}_k = z_k / n_k$

## ◆ Fehler

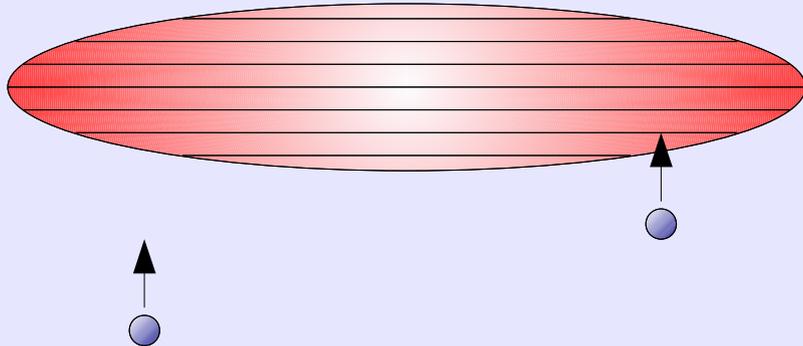
- ◆ Geschwindigkeitsbias
- ◆ Mittelung über Slotbreite
- ◆ Normierungsfehler
- ◆ Totzeit
- ◆ Datenrauschen
  
- ◆ Räumliche Ausdehnung des Messvolumens

## ◆ Korrektur

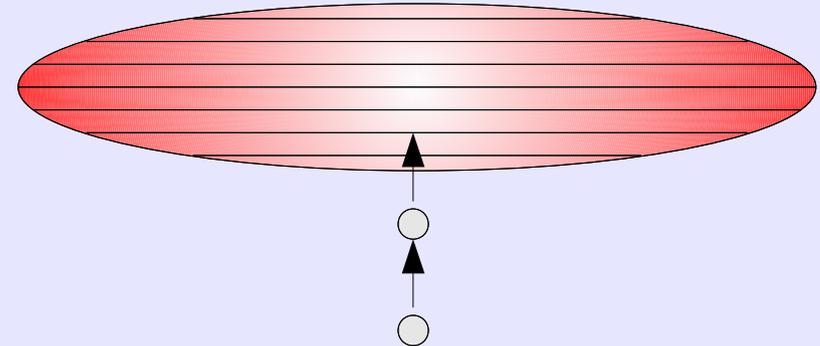
- ◆ Wichtung
- ◆ Fuzzy Slotting
- ◆ Local Normalization
- ◆ Local Time Estimation
- ◆ Re-Normierung + Modellanpassung

- ◆ Bias durch räumliche Ausdehnung des Messvolumens

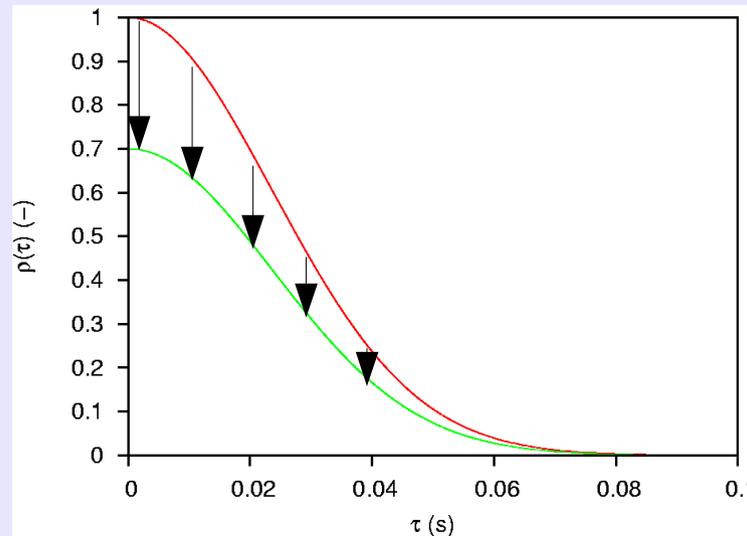
- ◆ Zwei kurz aufeinanderfolgende Teilchen mit großem Abstand quer zur Strömung



- ◆ Wertung wie zwei aufeinanderfolgende Teilchen ohne Abstand quer zur Strömung

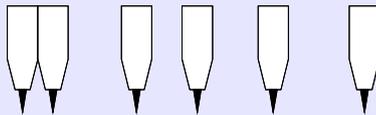


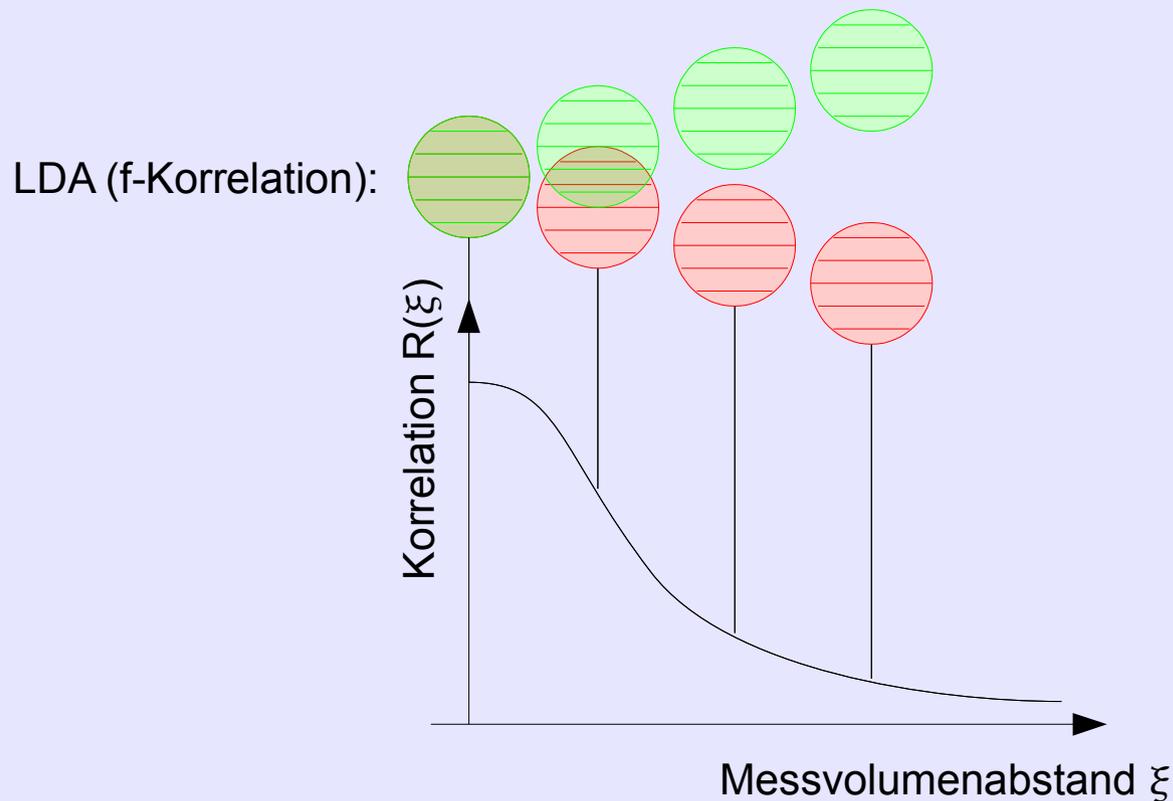
- ◆ Verringerung der f-Korrelation



- ◆ Auswirkung und Korrektur wie Datenrauschen

## ◆ Multi-Punkt-Messung

HDA (g-Korrelation): ? 



- ◆ Vorteil:
  - ◆ direkte Messung der räumlichen Korrelation
- ◆ Nachteile:
  - ◆ zwei oder mehr Sensoren
  - ◆ begrenzte räumliche Auflösung
  - ◆ zeitliche Korrelationen bleiben ungenutzt
  - ◆ hoher Justage- und Kalibrieraufwand
  - ◆ hoher zeitlicher Aufwand (mechanische Traversierung und wiederholte Messung)
  - ◆ hohe Anforderungen an die Langzeitstabilität
  - ◆ Probleme durch Rückwirkung der Sonden

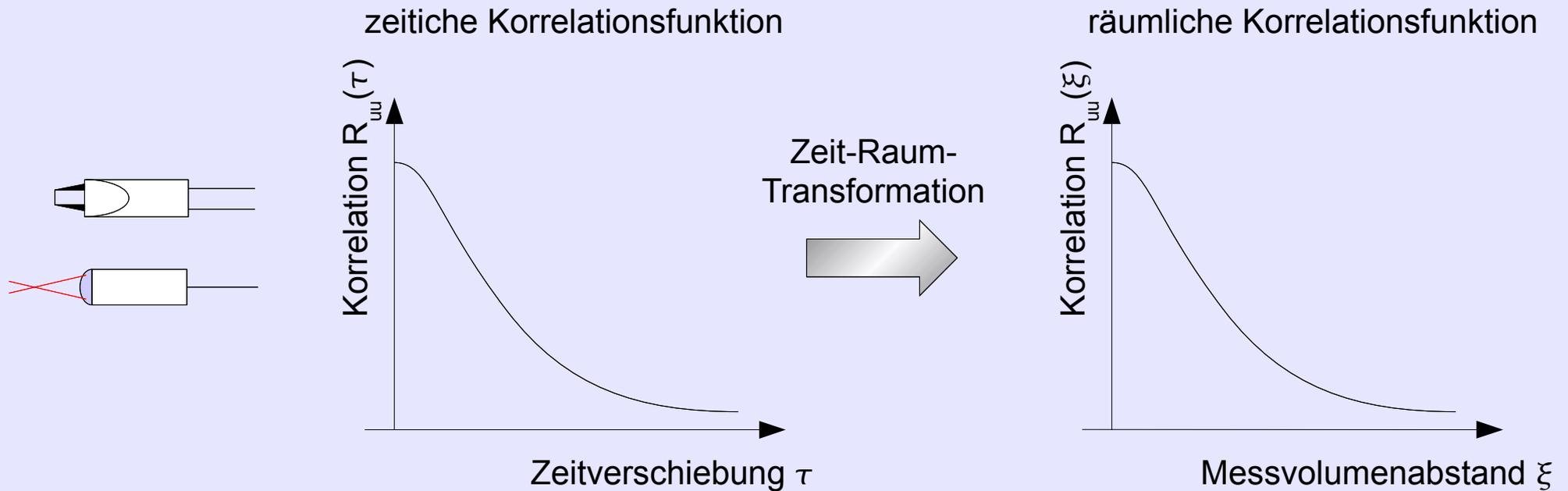
## ◆ Zeit-Raum-Transformation

### ◆ Vorteile:

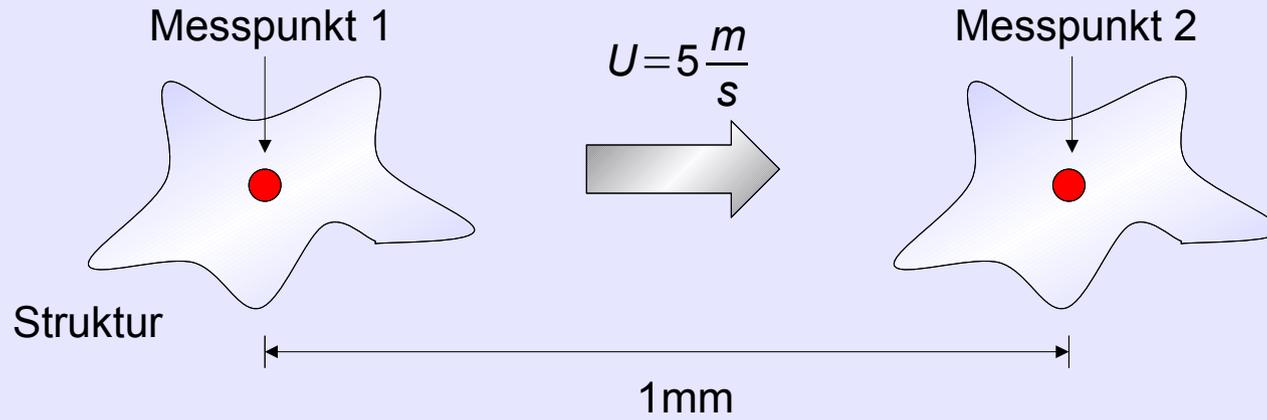
- ◆ schnelle Messwertaufnahme
- ◆ nur eine Sonde erforderlich
- ◆ hohe zeitliche und räumliche Auflösung
- ◆ einfache Bestimmung von Raum-Zeit-Korrelationen

### ◆ Nachteile:

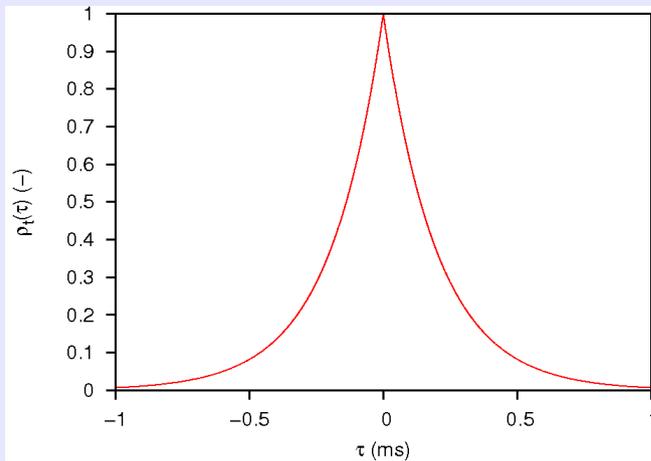
- ◆ Annahme von bestimmten Voraussetzungen
- ◆ nur Längskorrelationen (bei einkomponentiger Messung)



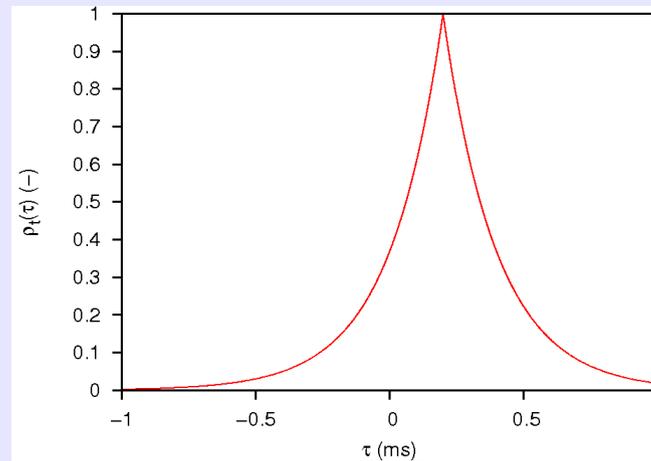
# ◆ Taylor-Hypothese



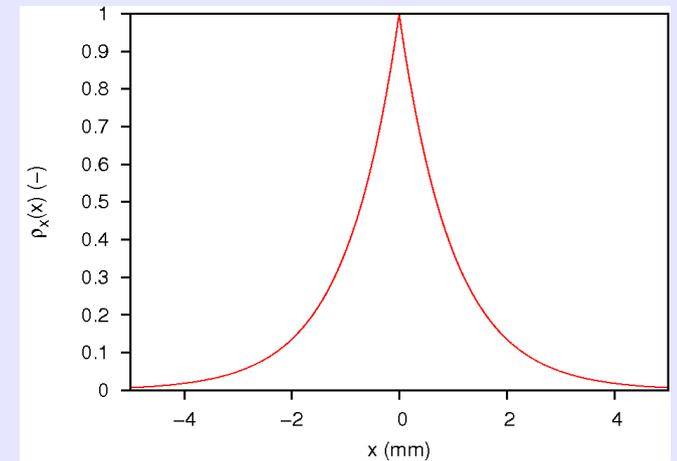
Zeitliche Autokorrelationsfunktion



Zeitliche Kreuzkorrelationsfunktion



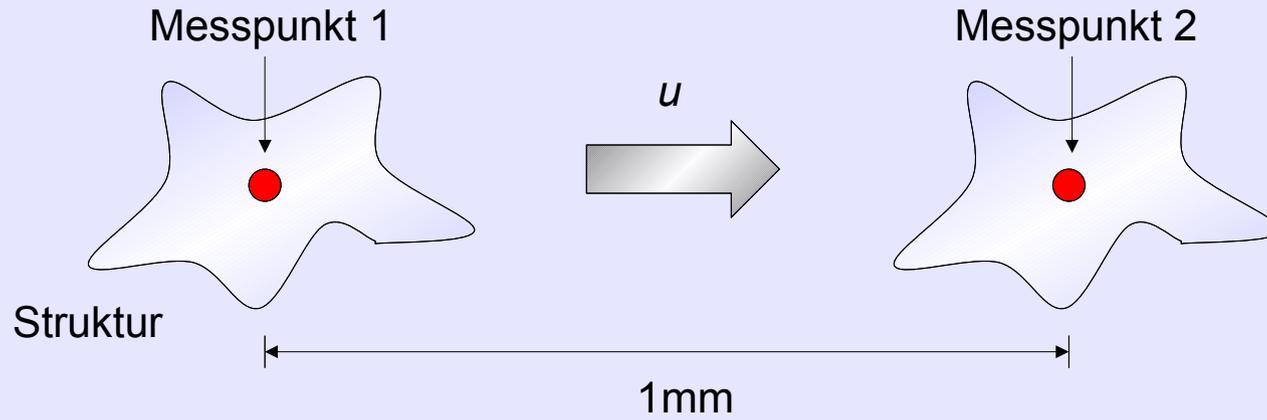
Räumliche Autokorrelationsfunktion



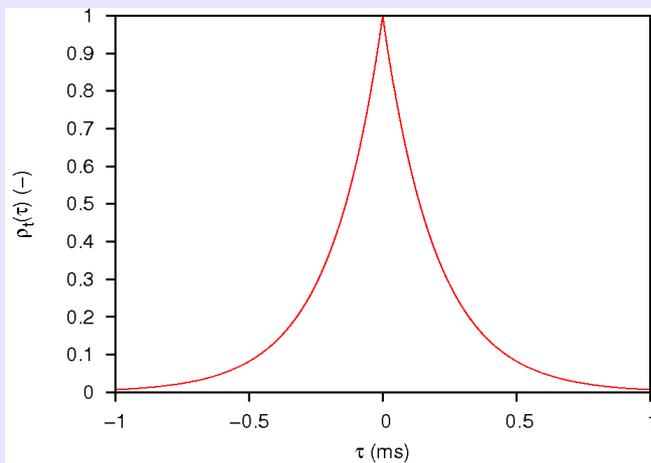
1ms  $\Rightarrow$  5mm

## Raum-Zeit-Transformation

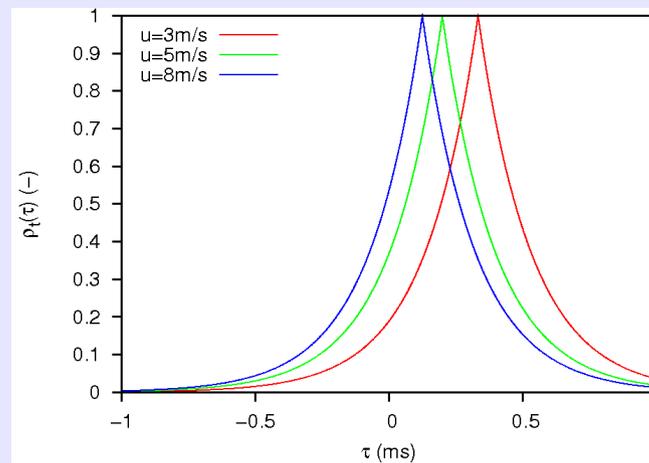
◆ veränderliche Geschwindigkeit



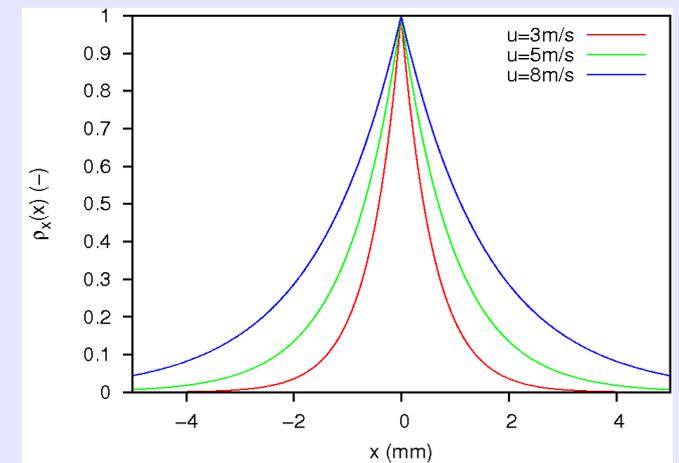
Zeitliche Autokorrelationsfunktion



Zeitliche Kreuzkorrelationsfunktion



Räumliche Autokorrelationsfunktion



$1\text{ms} \Rightarrow u \cdot 1\text{mm}$

Raum-Zeit-Transformation

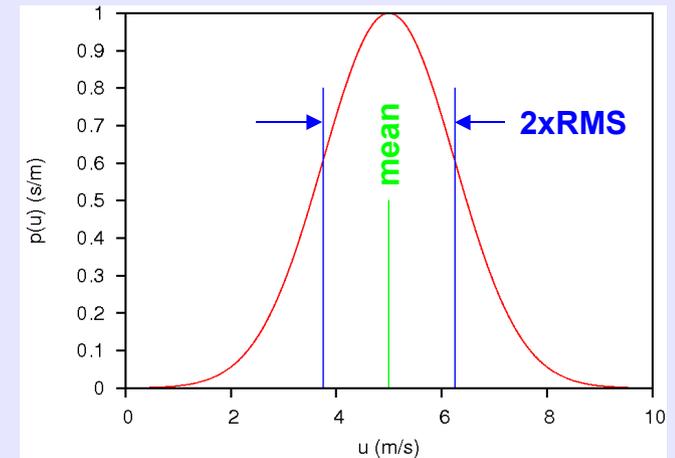
# ◆ Geschwindigkeitsstatistik

◆ für jedes u:

- ◆ Transformation
- ◆ Wahrscheinlichkeitsdichte

$$\rho(\xi) = \rho(\tau) \quad (\xi = u\tau)$$

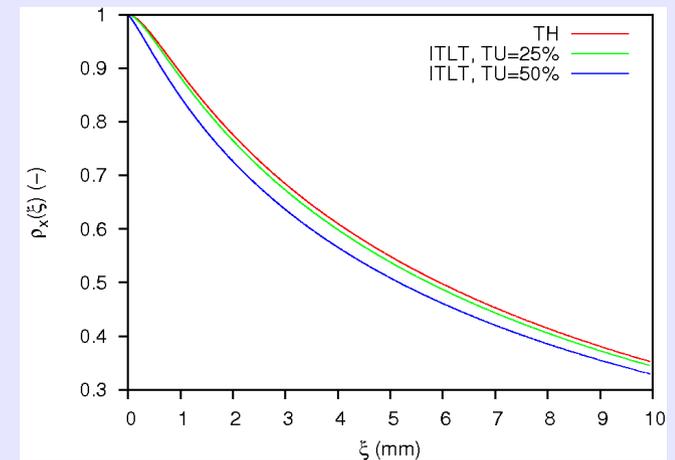
$$\rho(u)$$



◆ ganze räumliche Korrelationsfunktion:

- ◆ Integration über alle u

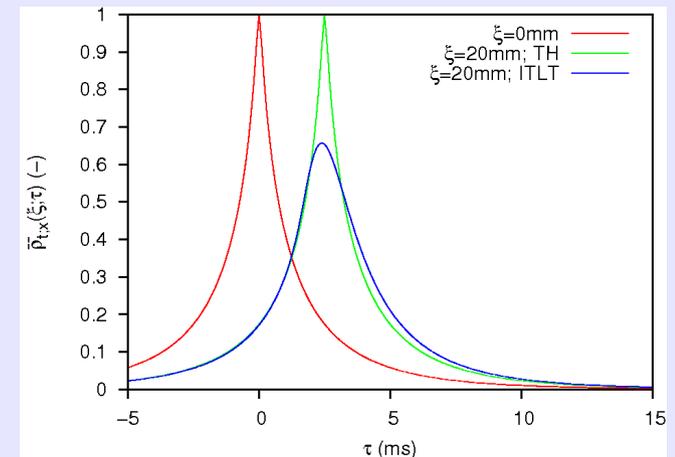
$$\bar{\rho}(\xi) = \int_{-\infty}^{+\infty} \rho(\xi)|_u \rho(u) du$$



- ◆ zusätzliche Zeitverschiebung für zweidimensionale Raum-Zeit-Korrelation

$$\bar{\rho}(\xi, \tau) = \int_{-\infty}^{+\infty} \rho(\xi, \tau)|_u \rho(u) du$$

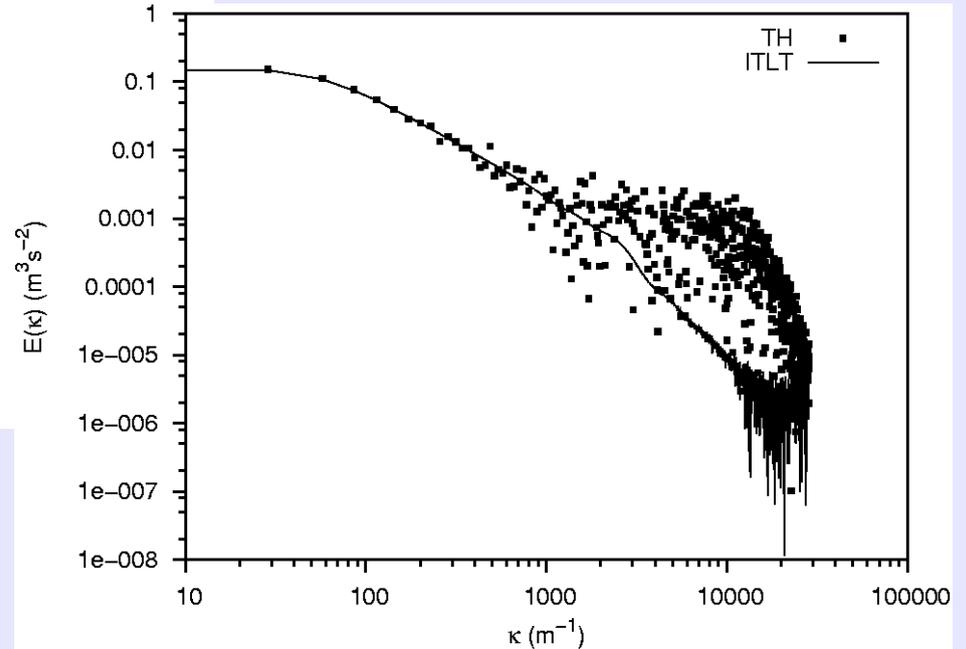
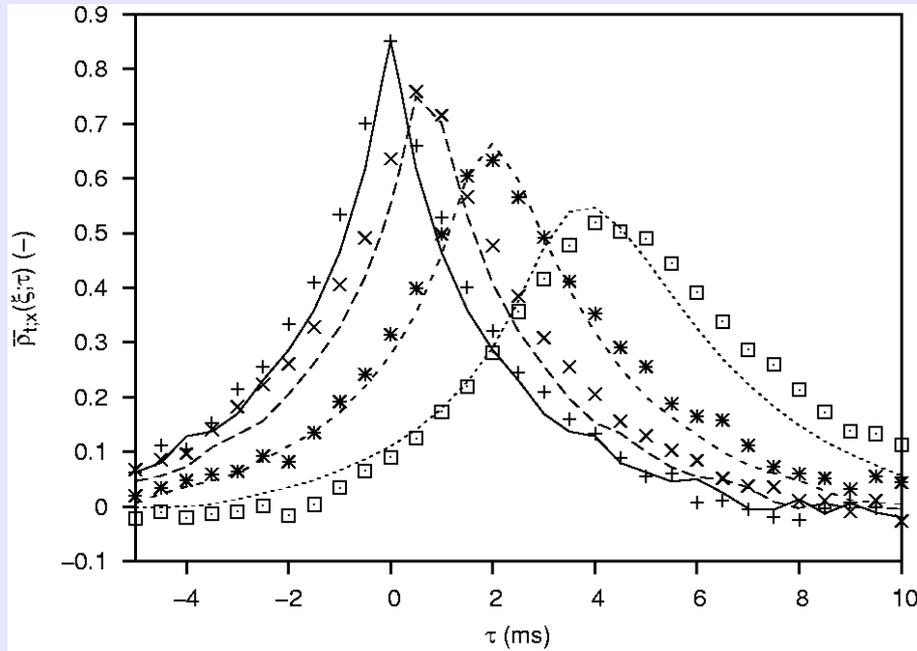
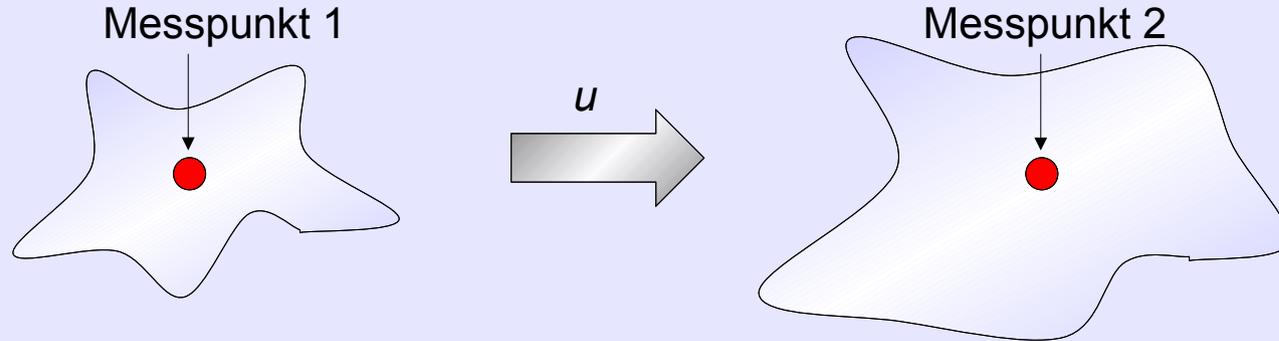
$$\rho(\xi, \tau)|_u = \rho(\tau + \xi/u)$$



## Raum-Zeit-Transformation

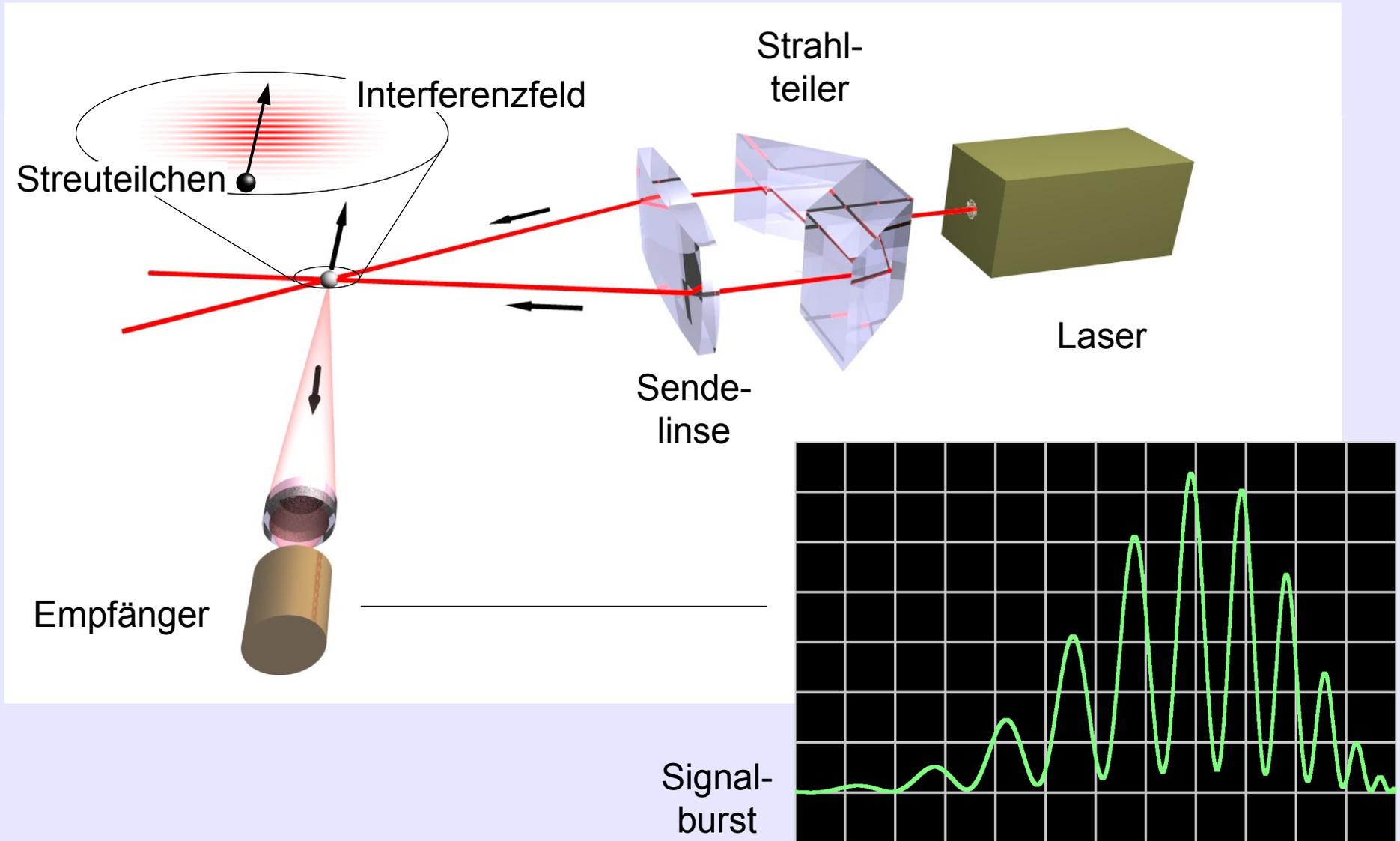


# ◆ Raum-Zeit-Korrelation

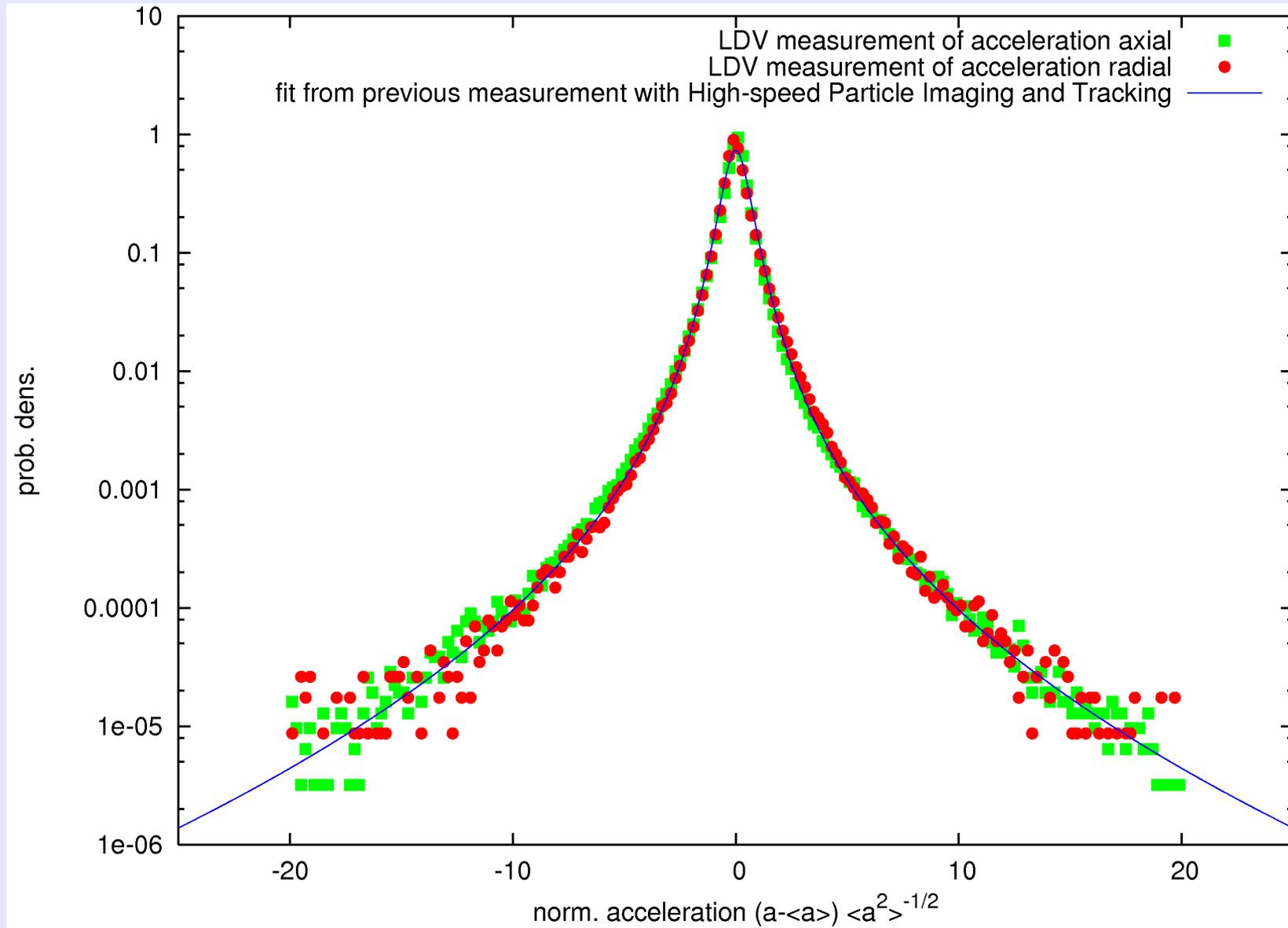


## Raum-Zeit-Transformation

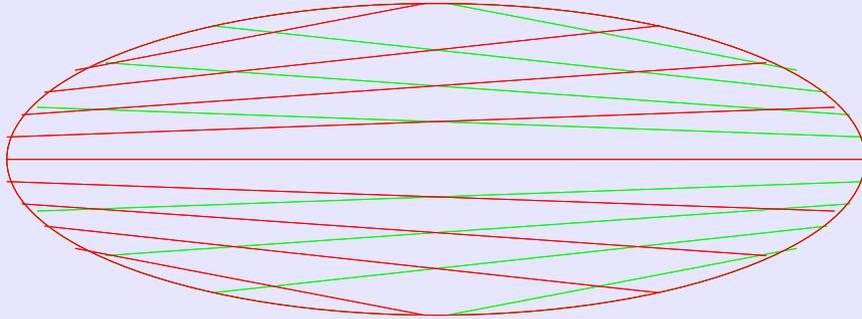
# LDA-Beschleunigungsmessung



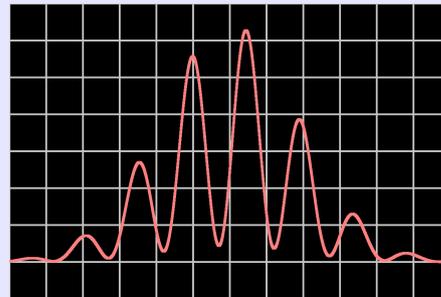
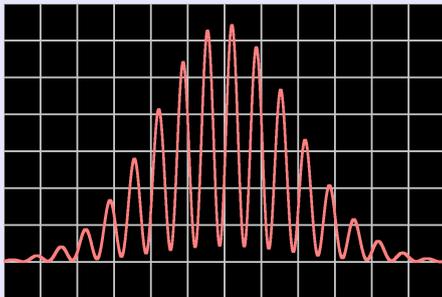
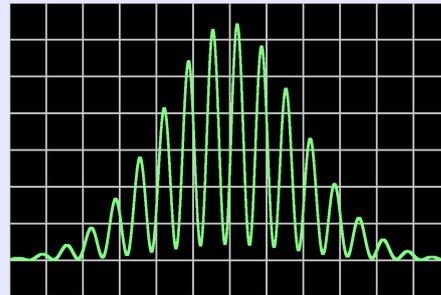
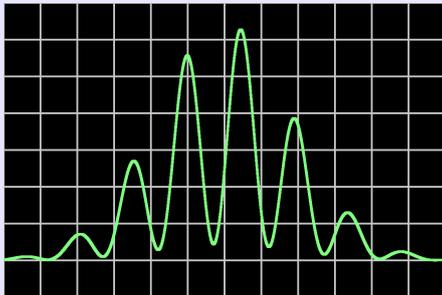
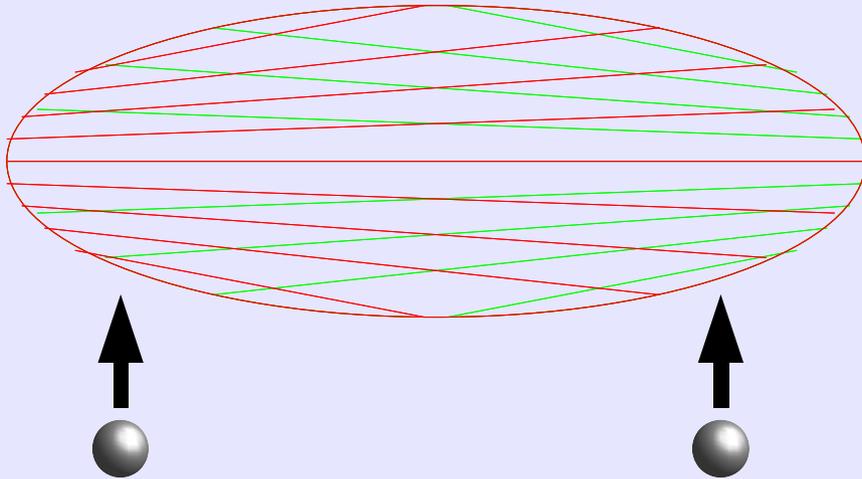
# LDA-Beschleunigungsmessung



# LDA-Profilsensor



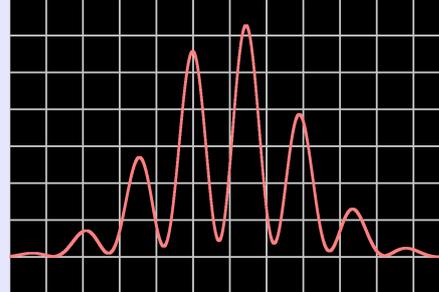
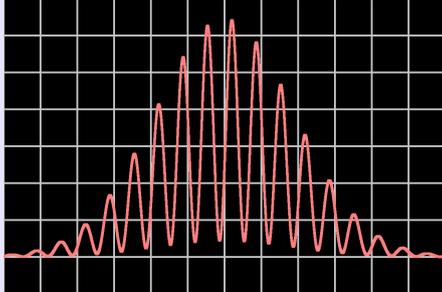
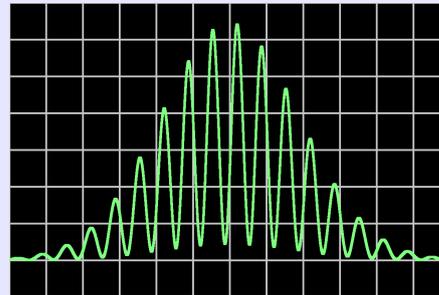
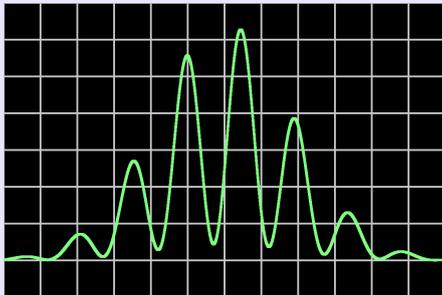
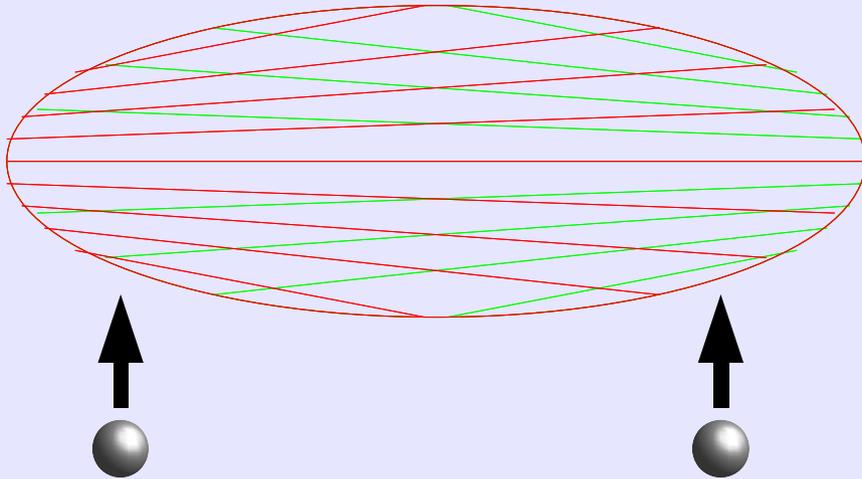
# LDA-Profilsensor



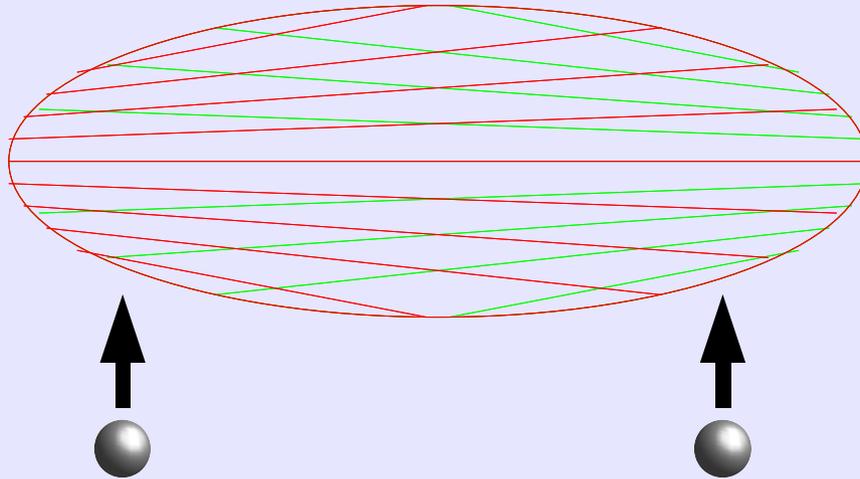
# LDA-Profilsensor

LDA-Datensatz

t      u      x



# LDA-Profilsensor

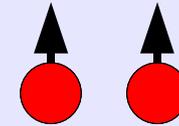
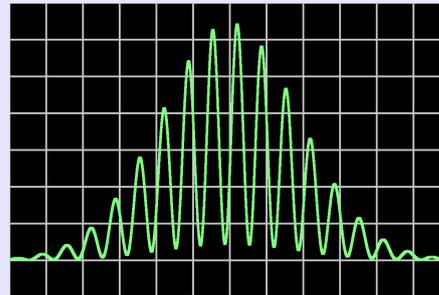
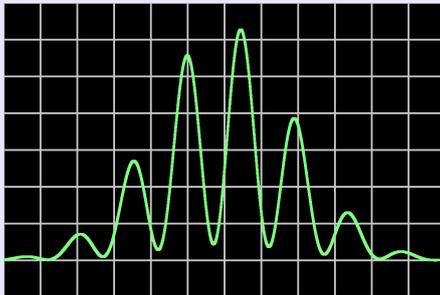


LDA-Datensatz

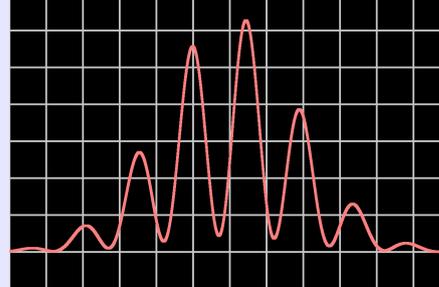
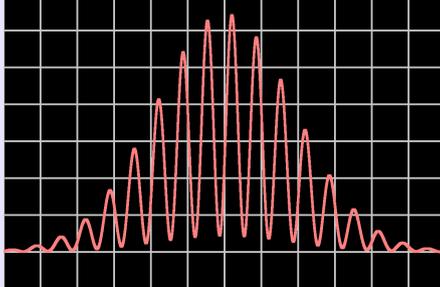
t      u      x



f-Korrelation  
durch Transformation

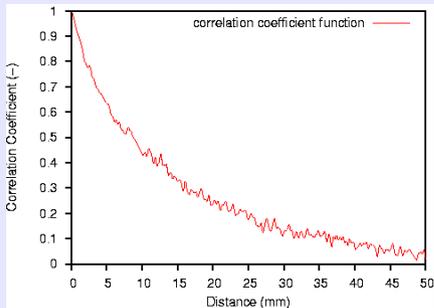
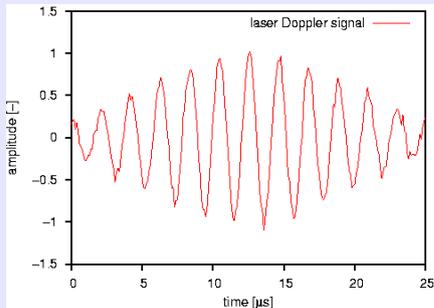
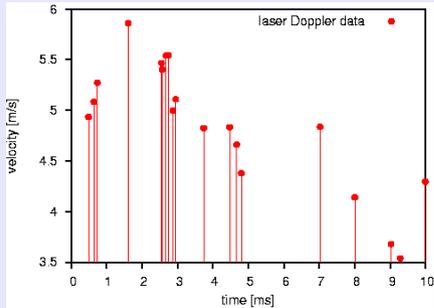


g-Korrelation  
durch x-Auflösung



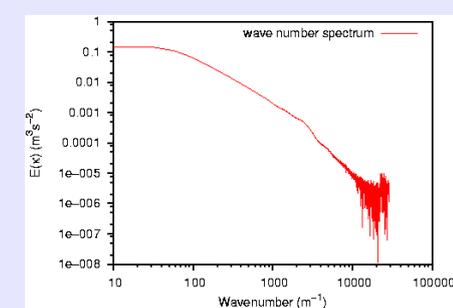
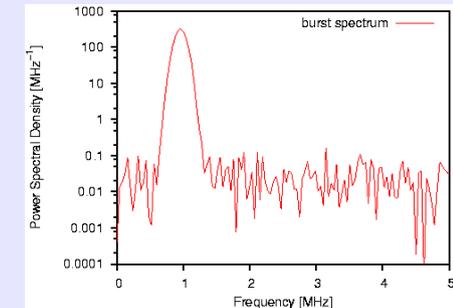
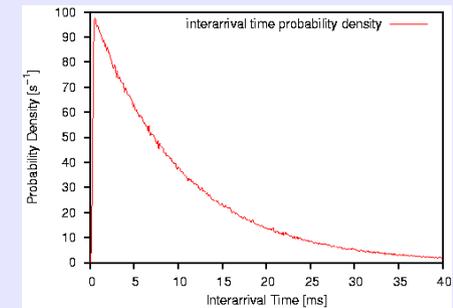
# Laser Doppler and Phase Doppler Signal and Data Processing Site

<http://ldvproc.nambis.de>



- ◆ News-Service
- ◆ Literatur

- ◆ LDA-Signalverarbeitung
- ◆ LDA-Datenverarbeitung
- ◆ Downloads
  - ◆ Simulationsprogramme
  - ◆ Experimentelle Referenzdaten (HDA und LDA)
  - ◆ Analyseprogramme



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