LDV signal / data processing

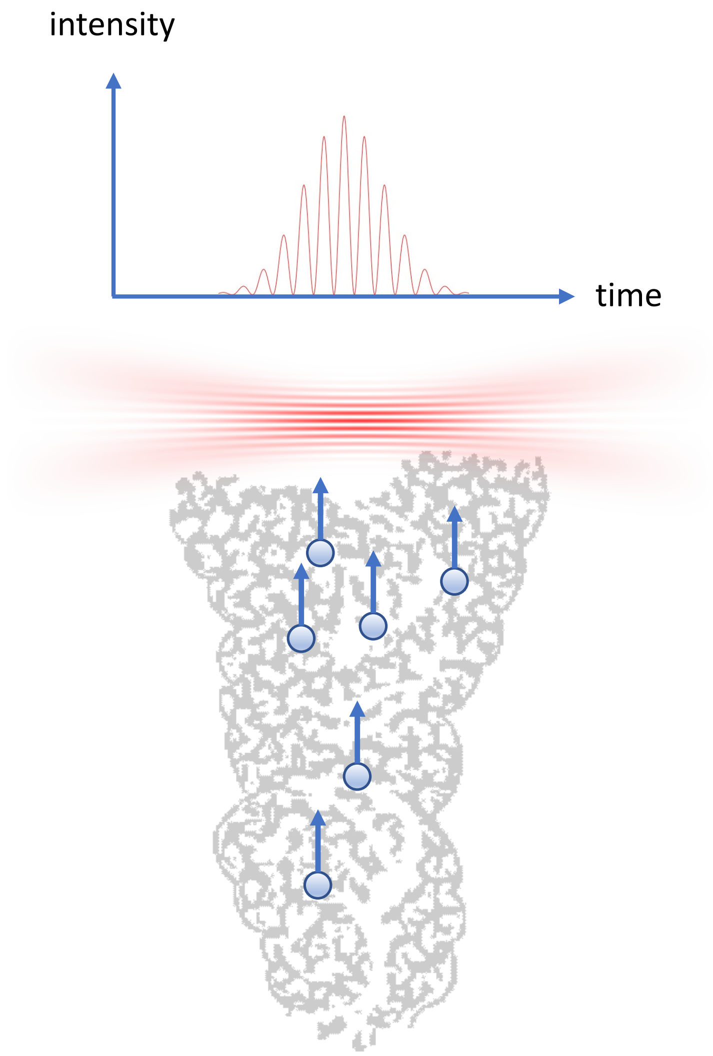
Holger Nobach

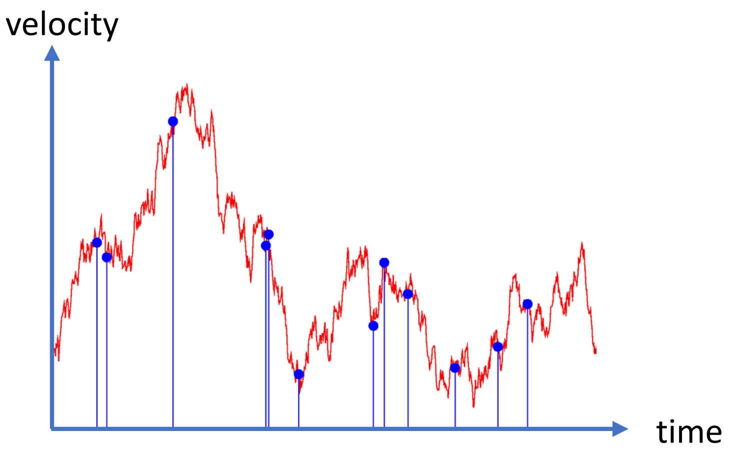
Max Planck Institute for Dynamics and Self-Organization

Göttingen

http://www.nambis.de/presentations/ldvbs2024

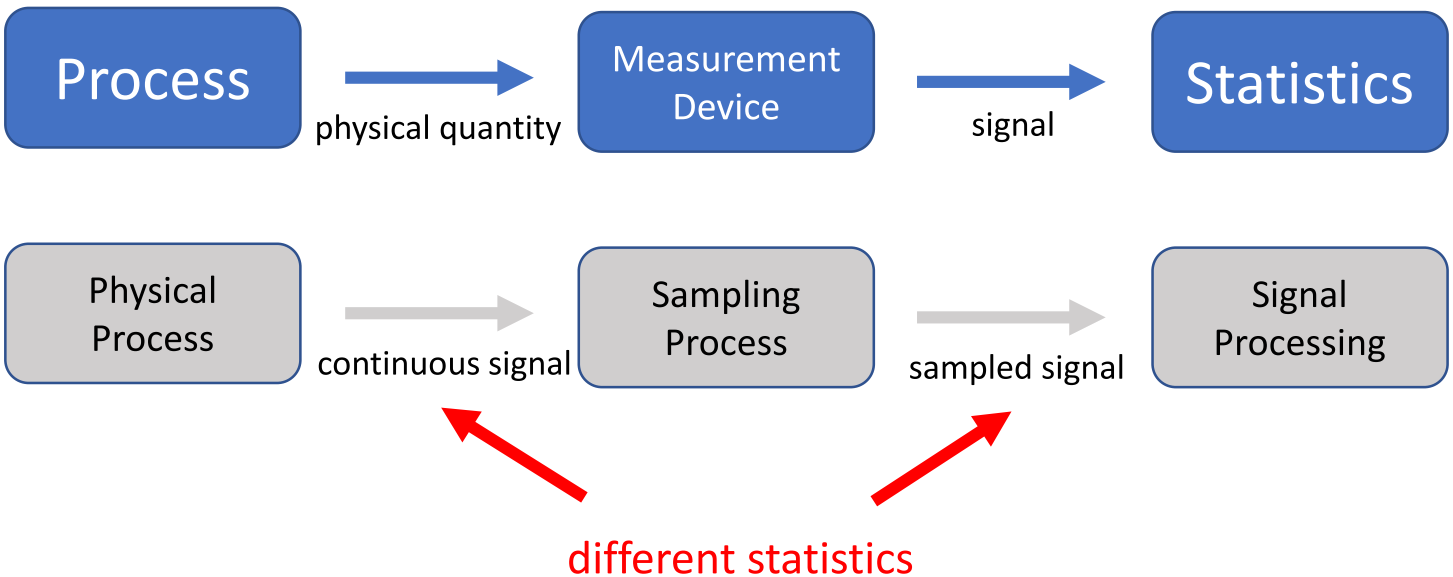
LDV signal or LDV data?



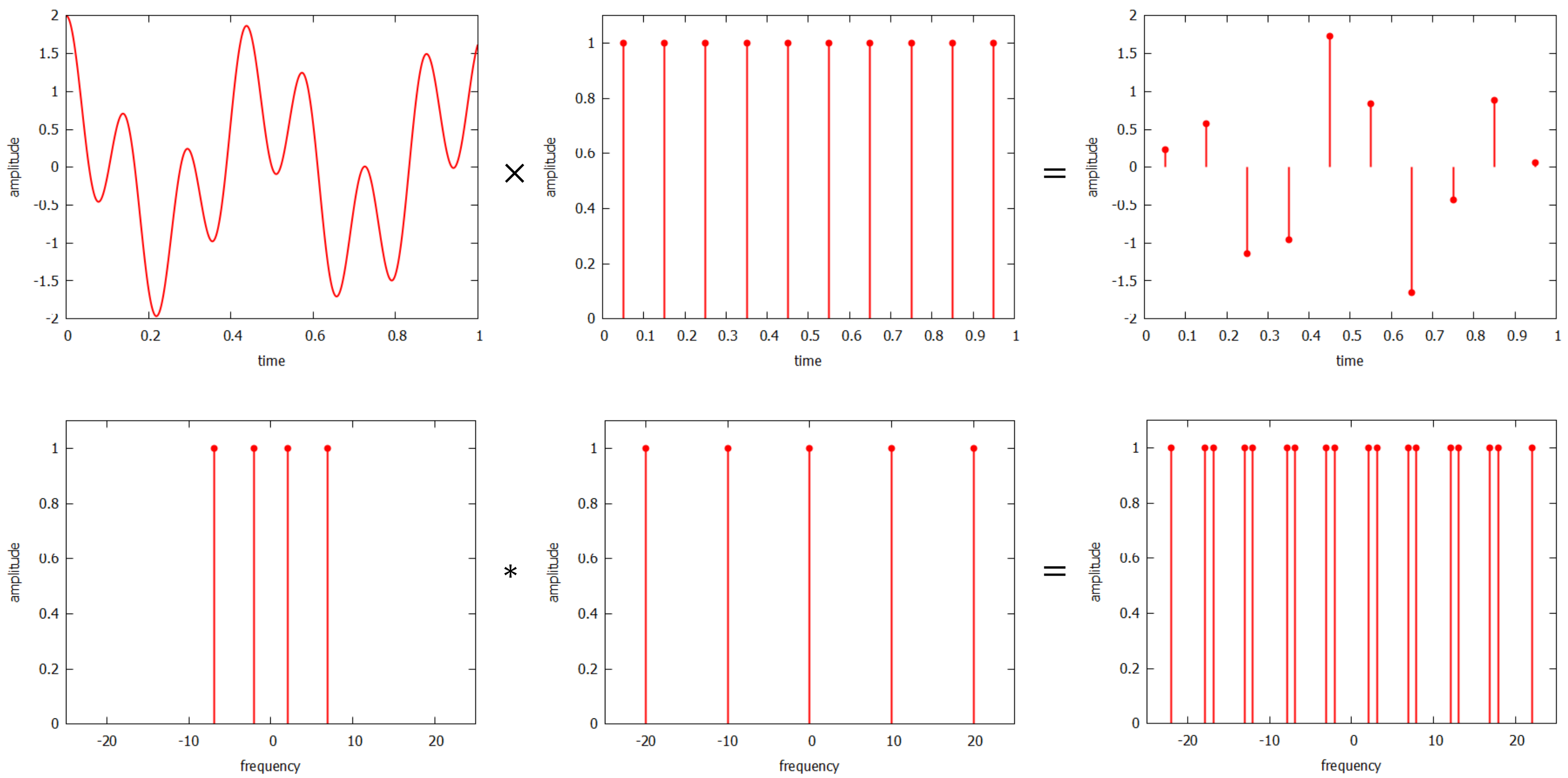


- unregularly sampled time series
- broad-band noise
- correlation between velocity and sampling rate
- processor dead time

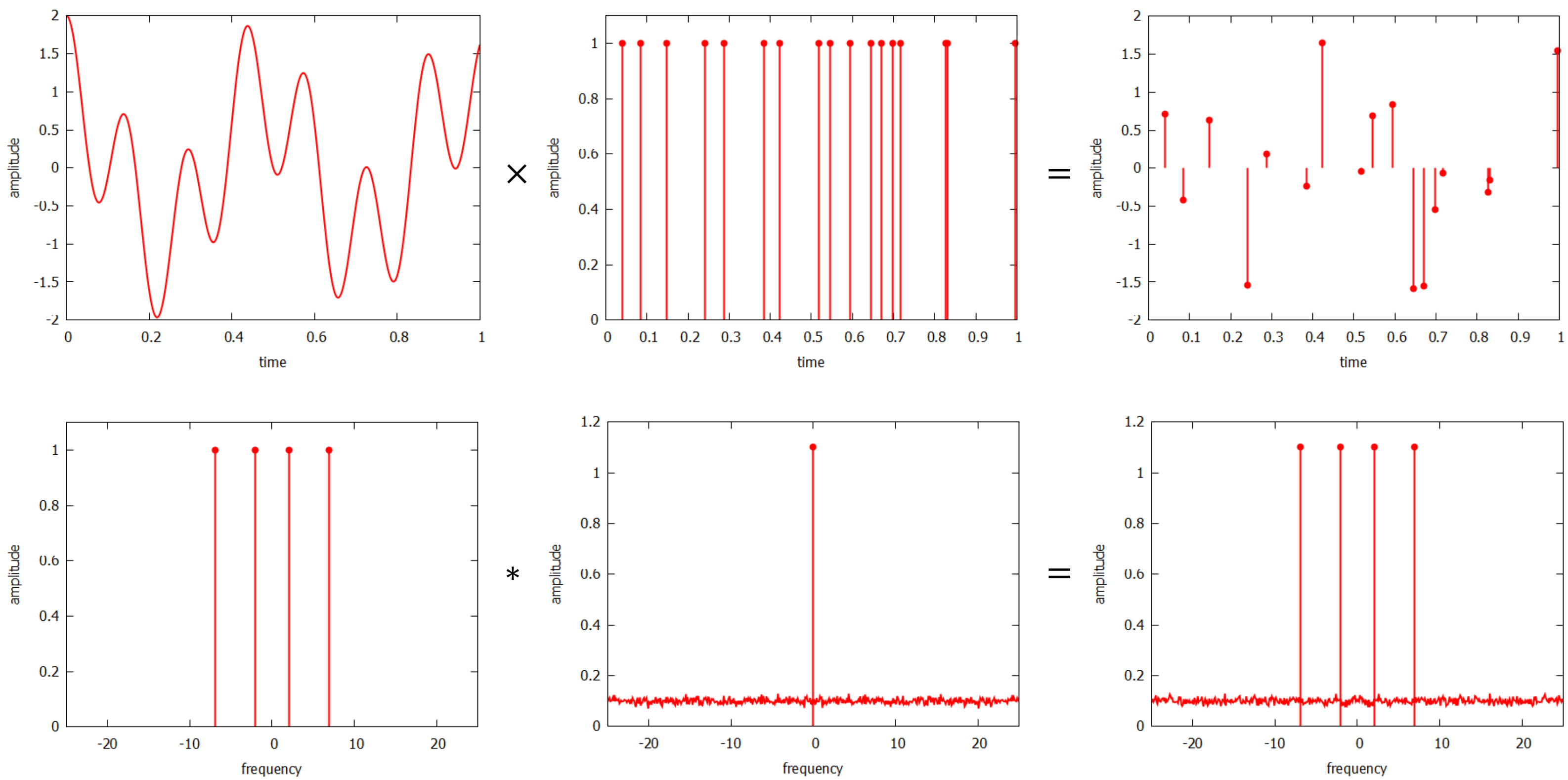
Decisions I


Process
physical quantity
Measurement Device
signal
Statistics
Physical Process
continuous signal
Sampling Process
sampled signal
Signal Processing
different statistics

Spectrum as an Example



Spectrum as an Example



Decisions II

Option 1

1. Analysing the sampling process
2. Adequate signal processing (considering the sampling process directly)

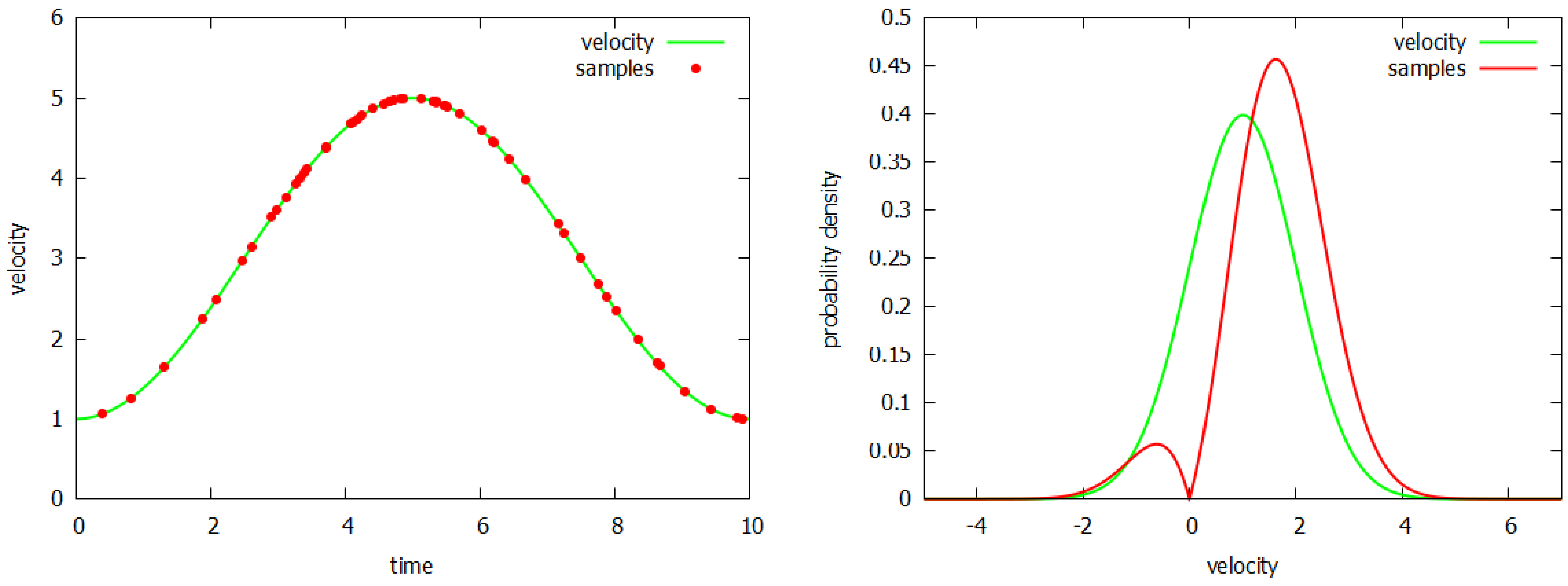
Option 2

1. Analysing the sampling process
2. Predicting the influence (bias)
3. Usual signal processing
4. Correcting the bias

Sources

- http://ldvproc.nambis.de
- A fair review of non-parametric bias-free autocorrelation and spectral methods for randomly sampled laser Doppler velocimetry. https://doi.org/10.1016/j.dsp.2018.01.018
- http://www.nambis.de/publications

Probability Density and Moments



Weighting

$$\bar{u} = \frac{\sum_{i=1}^{N} w_i u_i}{\sum_{i=1}^{N} w_i}$$

$$\sigma^2 = \frac{\sum_{i=1}^{N} w_i (u_i - \bar{u})^2}{\sum_{i=1}^{N} w_i}$$

- Velocity weighting:

$$w_i = \frac{1}{|u_i|}$$

… but 3D velocity and noise

- Inter-arrival time weighting:

$$w_i = t_i - t_{i-1}$$

… but at high data rates only

- Transit time weighting:

$$w_i = TT_i$$

Correlation and Spectrum Decisions III

Periodic Signal

Periodic Correlation Function

Fourier Transform ↓ ↑ Inverse Fourier Transform

Power Spectrum

Energy Signal

Energy Correlation Function

Fourier Transform ↓ ↑ Inverse Fourier Transform

Energy Density Spectrum

Non-Periodic Power Signal

Non-Periodic Correlation Function

Fourier Transform ↓ ↑ Inverse Fourier Transform

Power Density Spectrum

Correlation Function

$$R(\tau) = \langle u(t) \cdot u(t+\tau) \rangle$$

Slot Correlation

no self-products because of noise
and different statistics!

$$R(k\Delta\tau) = \frac{\sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} w_i w_j u_i u_j \left(|t_j - t_i - k\Delta\tau| < \frac{\Delta\tau}{2} \right)}{\sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} w_i w_j \left(|t_j - t_i - k\Delta\tau| < \frac{\Delta\tau}{2} \right)}$$

Fourier
Transform

Power Density Spectrum

Direct Spectrum

$$E(f) = \left| \int_{-\infty}^{\infty} u(t) \mathrm{e}^{-2\pi \mathbf{i} f t} \, \mathrm{d}t \right|^2 = \left| \sum_{i=1}^{N} u_i \, \mathrm{e}^{-2\pi \mathbf{i} f t_i} \right|^2$$

… but Energy Density Spectrum

… and Weighting?

… and Bias due to unregular sampling?

… and Self-products?

Direct Spectrum

$$E_u(f) = \left| \sum_{i=1}^{N} w_i u_i \, \mathrm{e}^{-2\pi \mathbf{i} f t_i} \right|^2 - \sum_{i=1}^{N} w_i^2 u_i^2 \quad \text{deconv.} \quad E_w(f) = \left| \sum_{i=1}^{N} w_i \, \mathrm{e}^{-2\pi \mathbf{i} f t_i} \right|^2 - \sum_{i=1}^{N} w_i^2$$

Inverse Fourier Transform ↓

Energy Correlation Function (of the sampled signal)

$R_u(\tau)$

Inverse Fourier Transform ↓

Energy Correlation Function (of the sampling signal)

$R_w(\tau)$

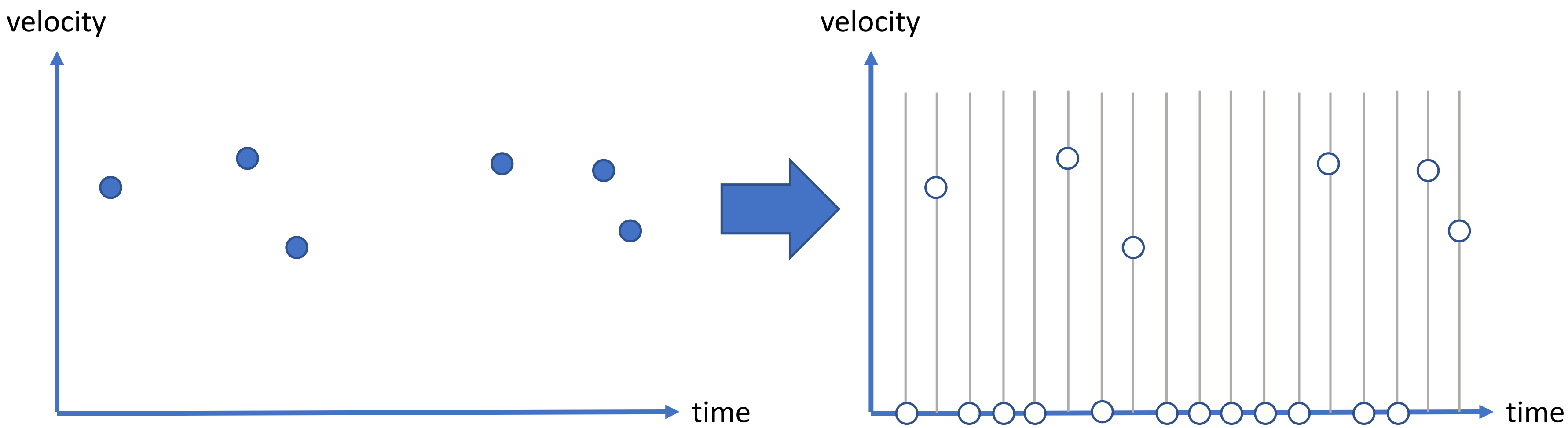
Correlation Function of the power process

$$R(\tau) = \frac{R_u(\tau)}{R_w(\tau)}$$

Fourier Transform ↓

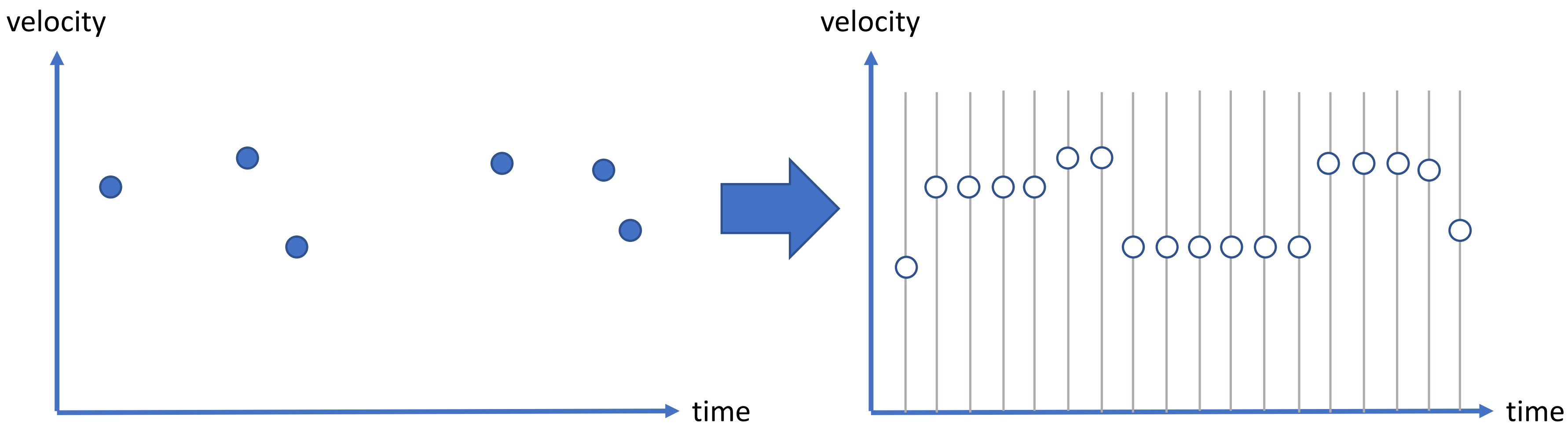
Power Density Spectrum

Time Quantization



Works similar to direct spectral estimation
… but faster due to FFT routines

Interpolation



Standard signal processing and later corrections
… but purely random sampling only and no weighting